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MODEL OF MAGNETIC DEGENERATE DWARF

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ABSTRACT. The macroscopic characteristics of the degenerate dwarves were calculated based on the equation of state of a spin-polarized electron gas at $T = 0K$ as well as at finite temperatures. It was shown that the spin-polarization cause an increase of the radius and mass in comparison with the characteristics of standard Chandrasekhar model. Within adopted model, the value of critical mass was estimated, by which an instability due to general relativity effects occurs. Parameters of model are: the relativistic parameter x_0 at stellar center, the chemical composition parameter $\mu_e = \langle A/Z \rangle$, the degree of spin-polarization ζ, the dimensionless temperature of core $T_0^* = k_B T_c/m_0 c^2$ and its radius $\xi_0 = R_c/R$ (R_c is the core radius, R is the radius of a dwarf). The inverse problem for a sample of massive dwarfs was solved using the known masses and radii and the parameters x_0, T_0^* were found for fixed values of spin-polarization.

Key words: degenerate dwarf, critical mass, relativistic parameter, spin-polarization.

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1. Introduction

The theory of the degenerate dwarfs was developed by S. Chandrasekhar in the 40-th years of the XX century, where the idea of R. Fowler's about mechanism of their stability and degeneration of electron subsystem [1, 2] was used. The Chandrasekhar model is two-component – it is a completely degenerate relativistic electron gas at absolute zero temperature and the static nuclear subsystem, which is considered as continuous classic medium. It has two parameters – the relativistic parameter $x_0 = (3\pi^2 n_0)^{1/3}\hbar/m_0 c$ (where n_0 is the number density of electrons in the stellar center of star, m_0 is the rest mass) and the chemical composition parameter $\mu_e = \langle A/Z \rangle$ (A is the mass number, Z is the number of protons in the nuclear)[3].

The standard Chandrasekhar model corresponds to non-magnetic massive dwarf, in which electron subsystem is in the paramagnetic state. At the same time from the observations are well known dwarfs with the masses which are very close to the Chandrasekhar limit $M_{ch} \approx 1.44 M_\odot$ or even exceed it [4]. Among these dwarfs can be stars with both weak and strong magnetic fields. The model of magnetic dwarf provides a simple interpretation of features of "super-Chandrasekhar" dwarfs and the existence of super-bright Ia type supernova.

Generalization of Chandrasekhar model for the case of finite temperature and consideration of magnetic field allows simultaneously to take into account the influence of these important factors on dwarfs. We constructed the model with five dimensionless parameters – the relativistic parameter x_0 at stellar center, the chemical composition parameter μ_e, the degree of spin-polarization ζ, the dimensionless core temperature $T_0^* = k_B T_c/m_0 c^2$ (the value $T_0^* = 1$ corresponds the temperature in stellar center $6.04 \cdot 10^9 K$) and the dimensionless core radius $\xi_0 = R_c/R$. Rough estimates of temperature of the degenerate dwarfs, which was obtained on the basis of their low luminosity, give the value of the order $10^6 \div 10^7$ K. Such factors, as the incomplete degeneration of electron subsystem, radiation pressure, and the thermal motion of nuclei, should be considered in the consistent theory of internal structure of these stars [5, 6].

In this work we consider model of magnetic dwarf at both absolute zero as well as finite temperature. We do not consider explicitly a magnetic field, which in the local approximation is assumed to be homogeneous. We restrict the consideration of the model, where the average occupation number of electrons $n_{\mathbf{k},s}(r)$ depends on the projection of the spin direction on the magnetic field [7].

2. The spin-polarized model at $T = 0K$

Let's consider an uniform model of the ideal relativistic degenerate electron gas, which consists of N electrons in the volume V influenced by a stable external magnetic field in the thermodynamic limit: $N, V \to \infty$, $N/V = const$.

In this model exists one selected direction, namely the direction of the vector of magnetic field intensity, and we assume that $n_{\mathbf{k},1/2} > n_{\mathbf{k},-1/2}$.

The value

$$\zeta_{\mathbf{k}} = \frac{1}{n_{\mathbf{k}}}(n_{\mathbf{k},1/2} - n_{\mathbf{k},-1/2}) \qquad (1)$$

determines a degree of polarization of the electron gas, where

$$n_{\mathbf{k}} = n_{\mathbf{k},1/2} + n_{\mathbf{k},-1/2}. \qquad (2)$$

From here we will assume, that the degree of spin-polarization does not depend on the wave vector,

$$\zeta = \frac{1}{n}(n_+ - n_-), \quad n = n_+ + n_-, \qquad (3)$$

where $n_\sigma = N_\sigma V^{-1}$ is the number density with given projection of the spin on the direction of the field. From the equation (3) we find, that

$$\begin{cases} n_+ = \dfrac{1}{2}\, n(1+\zeta), \\ n_- = \dfrac{1}{2}\, n(1-\zeta). \end{cases} \qquad (4)$$

From the normalization condition $\sum_{\mathbf{k},s} n_{\mathbf{k},s} = N$ we obtain the expressions for the Fermi wave numbers, which correspond to different projection of the spins:

$$\begin{aligned} &k_F^\sigma = k_F\lambda_\sigma, \quad \lambda_+ = (1+\zeta)^{1/3}, \\ &\lambda_- = (1-\zeta)^{1/3}, \quad k_F = (3\pi^2 n)^{1/3}. \end{aligned} \qquad (5)$$

The equation of state in the spatially two-component homogeneous case of the electron-nuclear model at $T = 0K$ has a parametric representation [7]:

$$\begin{aligned} &P = \sum_\sigma \frac{\pi m_0^4 c^5}{3h^3}\mathcal{F}(x_\sigma), \ \mathcal{F}(x_\sigma) = 4\int_0^{x_\sigma} \frac{dy\, y^4}{(1+y^2)^{1/2}}, \\ &\rho = m_u\mu_e \sum_\sigma n_\sigma = \frac{m_u\mu_e}{6\pi^2}\left(\frac{m_0 c}{\hbar}\right)^3 \sum_\sigma x_\sigma^3, \\ &\sigma = (+,-), \end{aligned} \qquad (6)$$

where x_σ is the relativistic parameter in our model.

To obtain the equation of state for inhomogeneous model, we should perform the replacement $x \to x(r), P_\sigma \to P_\sigma(r)$, $\rho \to \rho(r)$, $x_\sigma \to x_\sigma(r)$.

According to the formulae (5)

$$x_\sigma(r) = x(r)\lambda_\sigma, \qquad (7)$$

where $x(r) = \hbar k_F(r)(m_0 c)^{-1}$ corresponds to the Chandrasekhar model with paramagnetic subsystem.

Let us consider the mechanical equilibrium of star

$$\frac{dP(r)}{dr} = -G\,\rho(r)\frac{M(r)}{r^2}, \quad \frac{dM(r)}{dr} = 4\pi r^2\rho(r), \qquad (8)$$

where $P(r)$ is the pressure on the sphere of radius r, $\rho(r)$ is local density, $M(r)$ is a mass inside that sphere. The system of equations (8) is reduced to the nonlinear differential equation of second order for $x(r)$

$$\begin{aligned} &\frac{1}{r^2}\frac{d}{dr}\left\{r^2\left[\frac{\lambda_+^5}{\sqrt{1+x^2(r)\lambda_+^2}} + \frac{\lambda_-^5}{\sqrt{1+x^2(r)\lambda_-^2}}\right]\times \right. \\ &\left. \times\, x(r)\frac{dx}{dr}\right\} = -G(m_u\mu_e)^2\,\frac{64\pi^2 m_0^2 c^4}{3(hc)^3}x^3(r) \end{aligned} \qquad (9)$$

in which λ_+, λ_- are the parameters, mentioned above, moreover $\lambda_+^3 + \lambda_-^3 = 2$.

In a general case at arbitrary value of the parameter ζ, the equation (9) also can be reduced to the equation of the standard Chandrasekhar model using the substitution

$$\sum_{\sigma=\pm} \lambda_\sigma^3\{(1+x^2(r)\lambda_\sigma^2)^{1/2} - 1\} = \varepsilon_0^\zeta y(\xi), \ \xi = \frac{r}{\lambda}$$

where

$$\varepsilon_0^\zeta = \sum_{\sigma=\pm} \lambda_\sigma^3\{(1+x_0^2(r)\lambda_\sigma^2)^{1/2} - 1\}. \qquad (10)$$

To rewrite the right side of the equation (9) in terms of $y(\xi)$, let's determine $x(r)$ from the equation (10). We reduce this expression to the biquadratic equation

$$ax^4 - bx^2 + c = 0, \qquad (11)$$

where

$$\begin{cases} a = (\lambda_+^8 - \lambda_-^8)^2, \\ b(y) = 2\{(\lambda_+^8 + \lambda_-^8)[(\varepsilon_0^\zeta y)^2 + 4\varepsilon_0^\zeta y] + \\ \quad + 4(\lambda_+\lambda_-)^3(\lambda_+^5 + \lambda_-^5)\}, \\ c(y) = [(\varepsilon_0^\zeta y)^2 + 4\varepsilon_0^\zeta y]\{[(\varepsilon_0^\zeta y)^2 + 4\varepsilon_0^\zeta y] + \\ \quad + 4(\lambda_+\lambda_-)^3\}. \end{cases} \qquad (12)$$

The equation (11) has four real roots – two positive and two negative. The physical meaning have only positive roots. From them we choose that one, which in the limit $\zeta \to 0$(when $\lambda_+, \lambda_- \Rightarrow 1$) is a positive root of quadratic equation $c(y) - b(y)x^2 = 0$, because in this limit $a \Rightarrow 0$. Thus we find, that

$$x(r) = 2^{-1/2}(\lambda_+^8 - \lambda_-^8)^{-1}[b(y) - \varphi(y)]^{1/2}, \qquad (13)$$

where

$$\begin{aligned} &\varphi(y) = \{b^2(y) - 4ac(y)\}^{1/2} = 4(2+\varepsilon_0^\zeta y)(\lambda_+\lambda_-)^3\times \\ &\times\{(\lambda_+\lambda_-)^2[(\varepsilon_0^\zeta y)^2 + 4\varepsilon_0^\zeta y] + (\lambda_+^5 + \lambda_-^5)^2\}^{1/2}. \end{aligned} \qquad (14)$$

The equation (9) in a dimensionless form is

$$\begin{aligned} \frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{dy}{d\xi}\right) &= -\{\sqrt{2}(\lambda_+^8 - \lambda_-^8)^{-1}\times \\ &\times(\varepsilon_0^\zeta)^{-1}[b(y) - \varphi(y)]^{1/2}\}^3. \end{aligned} \qquad (15)$$

and satisfies the boundary conditions $y(0) = 1, y'(0) = 0$ and the condition $y(\xi) \geq 0$. The scale factor λ is determined by the expression

$$\frac{32\pi^2 G}{3(hc)^3}\left\{m_u\mu_e m_0 c^2\lambda\frac{\varepsilon_0^\zeta}{2}\right\}^2 = 1, \qquad (16)$$

which in the limit $\zeta \to 0$ coincides with the result of the standard model [3]. The dependence of the solutions of equation (15) on the relativistic parameter and the

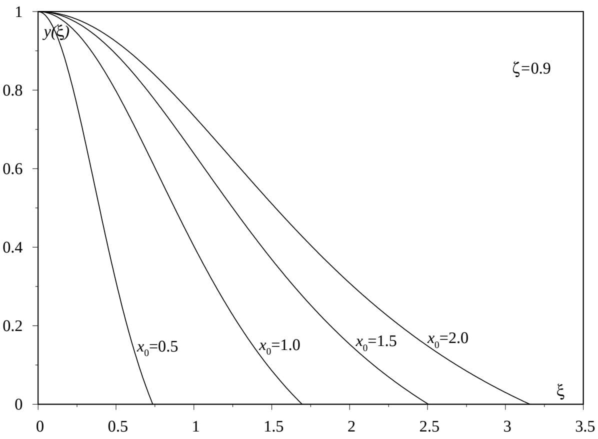

Figure 1: The solutions of equation (9) at fixed value $\zeta = 0.9$ for four values of x_0.

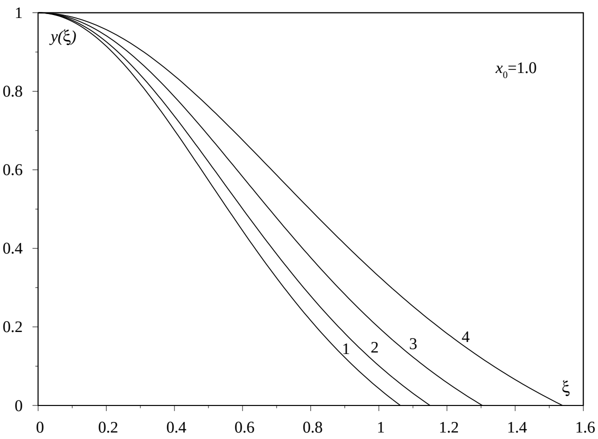

Figure 2: The solutions of equation (9) at fixed value $x_0 = 1.0$ ($\zeta = 0$ – curve 1, $\zeta = 0.2$ – curve 2, $\zeta = 0.4$ – curve 3, $\zeta = 0.6$ – curve 4)

.

degree of spin-polarization is illustrated in figures 1 and 2. The total mass of a star is the function of parameters x_0, μ_e, ζ:

$$M(x_0, \mu_e|\zeta) = \frac{M_0}{\mu_e^2}\mathcal{M}(x_0|\zeta),$$
$$\mathcal{M}(x_0|\zeta) = \xi_1^2(x_0|\zeta)\left|\frac{dy}{d\xi}\right|_{\xi=\xi_1(x_0|\zeta)} = 2.01824\cdots. \quad (17)$$

The radius of a star is determined by

$$R(x_0, \mu_e|\zeta) = \lambda\xi_1(x_0|\eta) = 2R_0\frac{\xi_1(x_0|\zeta)}{\mu_e\varepsilon_0^\zeta}. \quad (18)$$

Here

$$R_0 = \left(\frac{3}{2}\right)^{1/2}\frac{1}{4\pi}\left(\frac{h^3}{cG}\right)^{1/2}\frac{1}{m_0 m_u} = 1.12\cdot 10^{-2}R_\odot,$$
$$M_0 = \left(\frac{3}{2}\right)^{1/2}\frac{1}{4\pi}\left(\frac{hc}{Gm_u^2}\right)^{3/2}m_u = 2.89M_\odot \quad (19)$$

are the scale factors, $\xi_1(x_0|\zeta)$ is the dimensionless radius of star, which corresponds to the condition $y(\xi_1(x_0|\zeta)) = 0$.

The "mass-radius" relations obtained in the standard (solid curve) and spin-polarized models at different values of the parameter ζ are shown in fig. 3.

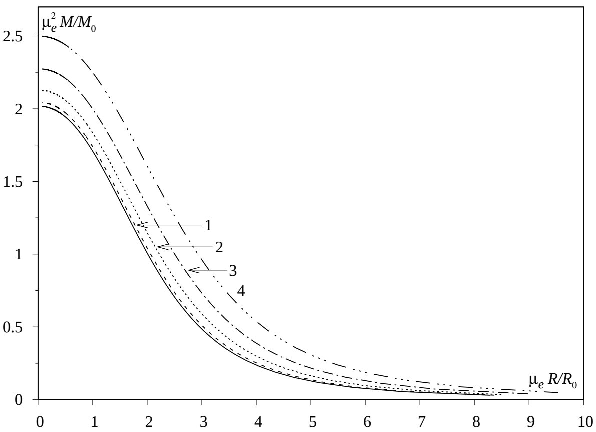

Figure 3: "Mass-radius" relation at different value ζ ($\zeta = 0$ – solid curve, $\zeta = 0.2$ – curve 1, $\zeta = 0.4$ – curve 2, $\zeta = 0.6$ – curve 3, $\zeta = 0.8$ – curve 4)

.

3. Influence of finite temperature effects on the spin-polarized model

In this section we briefly consider a model, which differs from the standard Chandrasekhar model by simultaneous accounting of magnetic field and finite temperature effects in the ideal relativistic electron systems. Here a dwarf is considered as spherically symmetric object, consisting of two regions – isothermal core with temperature T_0^* and radius R_c (occupies almost all volume of star) and a thin surface region, where the radial temperature distribution is approximated in the form $T^*(r) = T_0^*(\sqrt{1+x^2(r)}-1)\varepsilon_0^{-1}(x_c)$. By the averaging of temperature throughout the volume of a dwarf, as it is shown in [5, 6], the equation of model can be approximately represented in the reduced form:

$$P_{red} = \sum_\sigma \mathcal{F}_\sigma(r)\left\{1 + \frac{4}{3}\pi^2\left(\frac{T_0^*}{\varepsilon_0(x_0)}\right)^2\cdot S(x_0, \xi_0)\right\}, \quad (20)$$

where

$$S(x_0, \xi_0) = \varepsilon_0(x_0)\left\{\int_0^R dr\, r^2 x^3(r)\right\}^{-1}\times$$
$$\times\left\{\int_0^{R_c} dr\, r^2 f(x(r)) + \int_{R_c}^R dr\, r^2\times\right. \quad (21)$$
$$\left.\times f(x(r))\frac{(\sqrt{1+x^2(r)}-1)^2}{\varepsilon_0^2(x_c)}\right\},$$

$$
\begin{aligned}
&f(x(r)) = \frac{x^4(r)[r + x^2(r)][\sqrt{1+x^2(r)} - 1]^2}{\mathcal{F}(x(r))},\\
&\varepsilon_0(x_c) = \sqrt{1+x_c^2} - 1, \quad x_c = x(R_c),\\
&\mathcal{F}(x(r)) = x(r)(2x^2(r) - 3)\sqrt{1+x^2(r)} +\\
&+ 3\ln[x(r) + \sqrt{1+x^2(r)}].
\end{aligned}
$$

Here $\mathcal{F}(x(r))$ is the contribution of the ideal degenerate relativistic electron gas, R is the radius and $\xi_0 = R_c/R$ is the dimensionless radius of star, x_c is the value of the relativistic parameter on the edge of the isothermal core. Here the averaging is carried out using the mechanical equilibrium equation solutions at $T = 0K$ [3], and contribution to the pressure of finite temperature effects is considered as a correction. Using the reduced equation (20) and the dimensionless variables (10), the equation of state can be rewritten in the form, which was obtained in the previous section

$$
\begin{aligned}
\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{dy}{d\xi}\right) &= -\{\sqrt{2}(\lambda_+^8 - \lambda_-^8)^{-1}\times\\
&\times(\varepsilon_0^\zeta)^{-1}[b(y) - \varphi(y)]^{1/2}\}^3.
\end{aligned} \tag{22}
$$

However, the new scale λ is determined by:

$$
\frac{32\pi^2 G}{3(hc)^3}\left\{m_u\mu_e m_0 c^2\lambda\frac{\varepsilon_0^\zeta}{2}\right\}^2 = \eta, \tag{23}
$$

where

$$
\eta = 1 + \frac{4}{3}\pi^2\left(\frac{T_0^*}{\varepsilon_0(x_0)}\right)^2 \cdot S(x_0, \xi_0). \tag{24}
$$

Since the equilibrium equation does not contain the temperature as a parameter, the macroscopic characteristics of a star, such as mass and radius, can easily be rewritten in the following form:

$$
\begin{aligned}
R(x_0, \mu_e, \zeta|\xi_0, T_0^*) &= R_0\frac{\xi_1(x_0|\zeta)}{\mu_e\varepsilon_0^\zeta}\eta^{1/2},\\
M(x_0, \mu_e, \zeta|\xi_0, T_0^*) &= \frac{M_0}{\mu_e^2}\mathcal{M}(x_0|\zeta)\eta^{3/2}.
\end{aligned} \tag{25}
$$

Figures 4, 5 illustrate the dependence of mass and radius on the relativistic parameter x_0 at fixed values of the polarization parameter ζ, the dimensionless radius ξ_0 and different values of the dimensionless core temperature T_0^*. The temperature effects become more significant with decreasing of the relativistic parameter, i. e. decreasing dwarf mass. As can be seen from the figures, our model is not appropriate for low-mass dwarfs, because in such stars the temperature effects can not be considered as a correction. Therefore the branches of curves for which a mass are monotonically decreasing function of the relativistic parameter are non-physical. On the other hand in the region of large values of relativistic parameter, i. e. for massive dwarfs, the temperature effects are not significant.

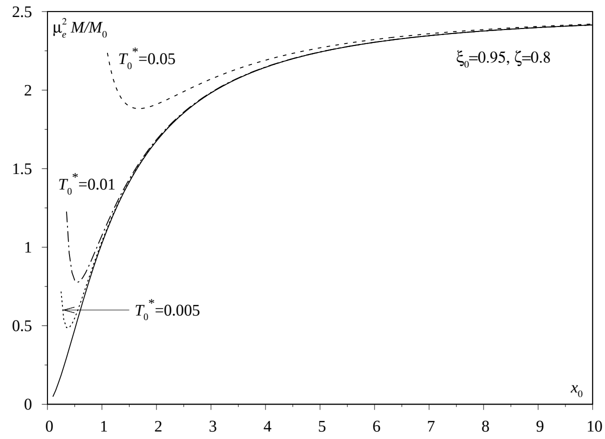

Figure 4: Dependence of dwarf's mass M on the relativistic parameter x_0 at fixed value of the dimensionless radius ξ_0 and the polarization parameter ζ for different values of the dimensionless temperature T_0^* (solid curve – $T_0^* = 0$).

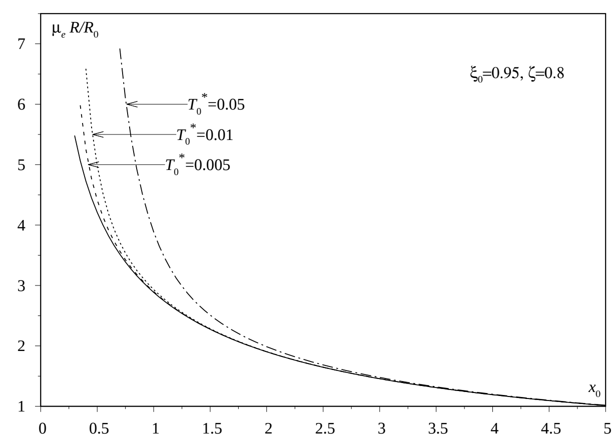

Figure 5: Dependence of dwarf's radius R on the relativistic parameter x_0 at fixed value of the dimensionless radius ξ_0 and the polarization parameter ζ for different values of the dimensionless temperature T_0^* (solid curve – $T_0^* = 0$).

We have used the sample of white dwarfs of DA spectral type from SDSS DR4 [8] and formed the subsample of the massive dwarfs with masses $M \geq 0.3M_0$ ($M \geq 0.87M_\odot$), because for such objects our model is appropriate (the temperature effects are small), on the other hand such objects can have strong enough magnetic fields.

Using the known values of masses and radii of the dwarfs, we can find from the system of equations (25) two parameters of the model x_0, T_0^*. At the same time other parameters are considered as free. The parameter μ_e is close to 2.0, we have taken the values from the work [5] for the same sample of objects. In the massive dwarfs the isothermal core occupies almost all volume of the star [6], so the parameter ξ_0 must be close to one. In this work, we have taken the value $\xi_0 = 0.99$.

The dependence of the dimensionless temperature T_0^* on the relativistic parameter x_0 for (as an example) five massive dwarfs at the values of the spin-polarization $\zeta = 0.10; 0.15; 0.20; 0.25; 0.30$ is shown in fig. 6. As can be seen from the figure, with increasing parameter ζ, i. e. models with stronger magnetic field, central temperature of dwarf considerably drops, and the relativistic parameter in its center slowly increases.

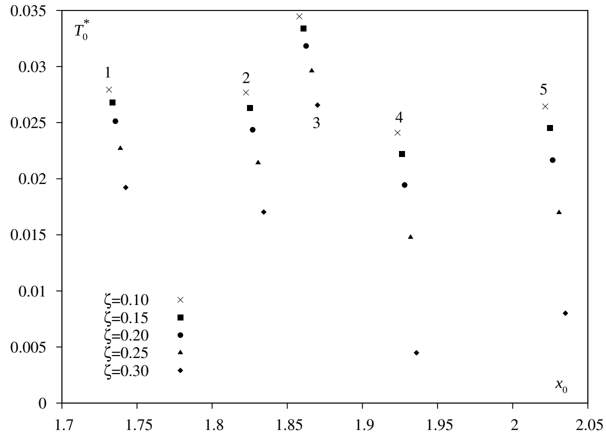

Figure 6: Dependence of the dimensionless temperature T_0^* on the relativistic parameter x_0 for the group of massive dwarfs at different values of the spin-polarization ζ (the dwarf 1 – $M = 0.870M_\odot$, $R = 0.0103R_\odot$, $T_{eff} = 38630K$; 2 – $M = 0.871M_\odot$, $R = 0.0094R_\odot$ $T_{eff} = 10330K$; 3 – $M = 0.871M_\odot$, $R = 0.0097R_\odot$, $T_{eff} = 19270K$; 4 – $M = 0.901M_\odot$, $R = 0.00963R_\odot$, $T_{eff} = 26120K$; 5 – $M = 0.931M_\odot$, $R = 0.0093R_\odot$, $T_{eff} = 27460K$).

In table 1 is shown the obtained data from the observations for the massive dwarfs (columns 2-4) [8], as well as determined parameters of the model [5], taking into account only incomplete degeneration of electron gas (columns 6-7), and considered here model, which also takes into account spin-polarization (columns 9-18). x_0^0 and $\tilde{\mu}_e$ are the values of the relativistic parameter in the stellar center and the average chemical composition parameter for the absolute cold white dwarf in the Chandrasekhar model (columns 5 and 8).

4. Critical mass and stability

For the massive dwarf at absolute zero temperature the relativistic parameter is sufficiently high ($x_0 \geq 10$). In the ultrarelativistic limit the equation of state of electron subsystem has a polytropic character,

$$P(r) \Rightarrow \frac{\pi m_0^4 c^5}{3h^3} \cdot x^4(r) \cdot \frac{(\lambda_+^4 + \lambda_-^4)}{2} \cdots . \tag{26}$$

Therefore for calculation of the dwarf's mass, taking into account the general relativity effects, we can use the approximate dependence between energy and mass of a star which is similar to that of Zeldovich and Novikov [9] for the paramagnetic state of electrons. For the case of our model

$$\begin{aligned} E \cong &\{AM - BM^{5/3}\}\rho_c^{1/3} + CM\rho_c^{-1/3} - \\ &- DM^{7/3}\rho_c^{2/3} - \frac{m_0 c^2}{\mu_e m_u} M, \end{aligned} \tag{27}$$

where M is the mass of dwarf, ρ_c is the central density. The first three terms in the right side of this equation correspond to Newton approximation, and the term $(-DM^{7/3}\rho_c^{2/3})$ approximately takes into account the contribution of general relativity effects. Here is used the following notations:

$$\begin{aligned} &A = k_1 K,\ B = k_2 G,\ C = k_3 \frac{m_0^2 c^3}{\hbar(\mu_e m_H)^{2/3}}, \\ &D = k_4 \frac{G^2}{c^2},\ K = \frac{3^{1/3}\pi^{2/3}}{4} \cdot \frac{\hbar c}{(m_u \mu_e)^{4/3}} \cdot \frac{(\lambda_+^4 + \lambda_-^4)}{2}, \\ &k_1 = 1.75579,\ k_2 = 0.639001, \\ &k_3 = 0.519723,\ k_4 = 0.918294. \end{aligned} \tag{28}$$

For convenience in the following calculations we rewrite energy and mass in dimensionless form (in units E_0, M_0)

$$\begin{aligned} &M = \frac{M_0}{\mu_e^2} \mathcal{M}(x_0|\zeta), \\ &E = \frac{E_0}{\mu_e^3} \mathcal{E}(x_0|\zeta), \quad E_0 = \frac{GM_0^2}{R_0}, \\ &\rho_c^{1/3} = x_0 \left(\frac{m_u \mu_e}{3\pi^2}\right)^{1/3} \frac{m_0 c}{\hbar} \equiv x_0 \delta. \end{aligned} \tag{29}$$

With dimensionless variables the formula (27) is transformed to the following form:

$$\begin{aligned} \mathcal{E}(x_0|\zeta) \cong &\left\{ \frac{k_1 (3\pi^2)^{1/3}}{4} \left(1 + \frac{2}{9}\zeta^2\right) \mathcal{M}(x_0|\zeta) - \right. \\ &\left. - k_2 \left(\frac{3\pi}{4}\right)^{1/3} \mathcal{M}^{5/3}(x_0|\zeta) \right\} \frac{x_0}{(3\pi^2)^{1/3}} + \\ &+ \frac{k_3}{x_0} \mathcal{M}(x_0|\zeta)\,(3\pi^2)^{1/3} - \\ &- \frac{k_4\, x_0^2}{(4\pi)^{2/3}} \frac{m_0}{m_u \mu_e} \mathcal{M}^{7/3}(x_0|\zeta) - \mathcal{M}(x_0|\zeta). \end{aligned} \tag{30}$$

From the equilibrium condition $\frac{\partial}{\partial x_0}\mathcal{E}(x_0|\zeta) = 0$ we find biquadratic equation for mass as a function of the relativistic parameter at small values ζ:

$$\begin{aligned} &\left(1 + \frac{2}{9}\zeta^2\right) \frac{k_1}{4} - \frac{k_2}{(4\pi)^{1/3}} \mathcal{M}^{2/3}(x_0|\zeta) - \frac{k_3\,(3\pi^2)^{1/3}}{x_0^2} - \\ &- \frac{k_4}{(2\pi^2)^{1/3}} \cdot \frac{x_0\, m_0}{m_u \mu_e} \mathcal{M}^{4/3}(x_0|\zeta) = 0. \end{aligned} \tag{31}$$

Table 1: The microscopic parameters and the macroscopic characteristics for the massive degenerate dwarfs of spectral type DA from the catalog SDSS DR4 [8]. Symbol "-" corresponds to the cases when no solutions found.

№	R/R_0	M/M_0	T_{eff}, K	x_0^0	$\tilde{x}_0$	$\tilde{T}_0^*$	$\tilde{\mu}_e$	$\zeta = 0.10$		$\zeta = 0.15$		$\zeta = 0.20$		$\zeta = 0.25$		$\zeta = 0.30$	
								x_0	T_0^*	x_0	T_0^*	x_0	T_0^*	x_0	T_0^*	x_0	T_0^*
295	0.74623	0.34989	34020	2.6079	2.3959	0.030982	2.0221	2.3915	0.030364	2.3953	0.027776	2.3973	0.023905	2.4026	0.017152	-	-
362	0.85193	0.30832	24440	2.0468	1.9322	0.021003	2.0218	1.9277	0.020514	1.9306	0.018215	1.9323	0.014622	1.9363	0.0071401	-	-
555	0.79354	0.3291	23240	2.296	2.1654	0.02363	2.0206	2.1609	0.022898	2.1643	0.020128	2.1661	0.015746	2.1709	0.005436	-	-
650	0.79354	0.3291	26140	2.296	2.1654	0.02363	2.0206	2.1609	0.022898	2.1643	0.020128	2.1661	0.015746	2.1709	0.005436	-	-
728	0.88187	0.30832	30090	2.0468	1.8528	0.026291	2.0218	1.8482	0.026253	1.8509	0.024739	1.8527	0.022552	1.8564	0.019157	1.8602	0.013811
780	0.8719	0.30139	18340	1.9732	1.861	0.020378	2.0225	1.8564	0.019971	1.8592	0.01784	1.861	0.014528	1.8647	0.0079835	-	-
944	0.86686	0.30485	18760	2.0095	1.883	0.021713	2.0221	1.8784	0.021361	1.8813	0.019327	1.883	0.016233	1.8868	0.010592	-	-
1001	0.77463	0.33603	25450	2.3914	2.25	0.024951	2.0207	2.2456	0.024155	2.2492	0.021273	2.251	0.016724	2.256	0.0062781	-	-
1005	0.8469	0.31178	18830	2.0851	1.9552	0.022433	2.0215	1.9507	0.022007	1.9536	0.019822	1.9554	0.016486	1.9594	0.010226	-	-
1025	0.83167	0.32217	27460	2.2074	2.0262	0.026704	2.0208	2.0217	0.026437	2.0248	0.024499	2.0266	0.021661	2.0309	0.016974	2.0351	0.0080077
1430	0.8567	0.31178	26120	2.0851	1.9279	0.024358	2.0215	1.9234	0.024103	1.9263	0.022221	1.9281	0.019438	1.932	0.01477	1.936	0.0044897
1451	0.77463	0.33603	27670	2.3914	2.25	0.024951	2.0207	2.2456	0.024155	2.2492	0.021273	2.251	0.016724	2.256	0.0062781	-	-
1513	0.82719	0.31871	22180	2.1654	2.0306	0.023264	2.021	2.0261	0.022763	2.0293	0.020428	2.031	0.016853	2.0353	0.0099731	-	-
1902	0.92357	0.30139	38630	1.9732	1.736	0.027763	2.0225	1.7314	0.027935	1.7338	0.026779	1.7356	0.025126	1.7389	0.022694	1.7425	0.019218
2003	0.8719	0.30139	20450	1.9732	1.861	0.020378	2.0225	1.8564	0.019971	1.8592	0.01784	1.861	0.014528	1.8647	0.0079835	-	-
2237	0.85694	0.30485	13980	2.0095	1.9095	0.019557	2.0221	1.905	0.018998	1.9079	0.016558	1.9096	0.012618	1.9135	0.00084474	-	-
2383	0.82719	0.31871	23290	2.1654	2.0306	0.023264	2.021	2.0261	0.022763	2.0293	0.020428	2.031	0.016853	2.0353	0.0099731	-	-
2724	0.85193	0.30832	19960	2.0468	1.9322	0.021003	2.0218	1.9277	0.020514	1.9306	0.018215	1.9323	0.014622	1.9363	0.0071401	-	-
2836	0.89115	0.32217	58380	2.2074	1.8626	0.034161	2.0208	1.858	0.034468	1.8608	0.033375	1.8625	0.031834	1.8663	0.029604	1.8701	0.026559
2893	0.89208	0.30832	36820	2.0468	1.8271	0.027619	2.0218	1.8225	0.027679	1.8252	0.026317	1.827	0.024365	1.8306	0.021414	1.8343	0.017033
2914	0.7836	0.33603	28100	2.3914	2.2182	0.027283	2.0207	2.2138	0.026729	2.2173	0.024275	2.2191	0.020582	2.224	0.01392	-	-
3016	0.82656	0.32564	31420	2.2509	2.0505	0.028143	2.0207	2.0461	0.027924	2.0492	0.026047	2.051	0.023322	2.0554	0.018923	2.0596	0.011319
3035	0.882	0.30139	27420	1.9732	1.8352	0.022304	2.0225	1.8307	0.022068	1.8334	0.020252	1.8351	0.017537	1.8388	0.012921	-	-

in the Newton approximation for large relativistic parameter

$$\mathcal{M}_H(x_0|\zeta) \cong \left(1+\frac{2}{9}\zeta^2\right)^{3/2}\left(\frac{k_1}{k_2}\right)^{3/2}\frac{\pi^2}{4} - \\ -\left(1+\frac{2}{9}\zeta^2\right)^{1/2}\frac{k_3\,k_1^{1/2}}{k_2^{3/2}}\frac{6\sqrt{3}\pi^{7/6}}{x_0^2}+\cdots. \quad (32)$$

Taking into account the general relativity effects leads to a decrease of the dwarf's mass. In particular, for $\zeta = 0$ there is such value $x_0^{(0)}$ when $\frac{\partial}{\partial x_0}\mathcal{M}(x_0|0) = 0$. This value determines the critical mass allowed for white dwarf and it can be found as a root of the equation

$$\frac{2k_3\,(3\pi^2)^{1/3}}{x_0^3} - \frac{k_4}{(2\pi^2)^{1/3}}\cdot\frac{m_0}{m_u\mu_e}\mathcal{M}^{4/3}(x_0|0) = 0, \quad (33)$$

where $\mathcal{M}(x_0|0)$ is solution of the equation (31) at $\zeta = 0$. The instability of paramagnetic dwarf occurs in the point $x_0^{(0)}$. The critical mass of corresponding dwarf is less than the Chandrasekhar limit $\mathcal{M}_{ch} = \left(\frac{k_1}{k_2}\right)^{3/2}\frac{\pi^{1/2}}{4}$. In the case $\zeta \neq 0$ the solution

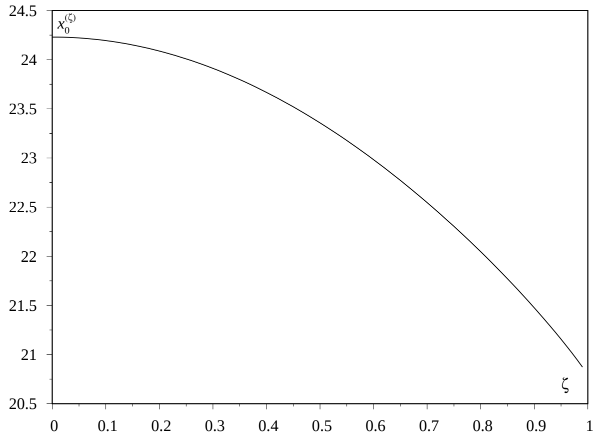

Figure 7: Dependence of the critical relativistic parameter $x_0^{(\zeta)}$ on the parameter ζ.

of equation (31) can exceed the Chandrasekhar limit and maximum mass (when the instability occurs) is achieved by some $x_0^{(\zeta)}$. The values $x_0^{(\zeta)}$ and $\mathcal{M}(x_0|\zeta)$ are determined by the equations (31) and

$$\frac{2k_2\,(3\pi^2)^{1/3}}{x_0^3} - \frac{k_4}{(2\pi^2)^{1/3}}\cdot\frac{m_0}{m_u\mu_e}\mathcal{M}^{4/3}(x_0|0) = 0. \quad (34)$$

The dependence of critical value of the relativistic parameter $x_0^{(\zeta)}$ and the dimensionless mass $\mathcal{M}(x_0|\zeta)$ on the parameter ζ are shown in figures 7, 8. As we can see, the consideration of spin-polarization of electron gas in the model of white dwarf structure can yield dwarfs with masses exceeding the Chandrasekhar limit up to 40%. The critical value of relativistic parameter in stellar center (i.e. central density) decreases with increasing polarization parameter and in the model with fully polarized electron subsystem is equal to 20.9.

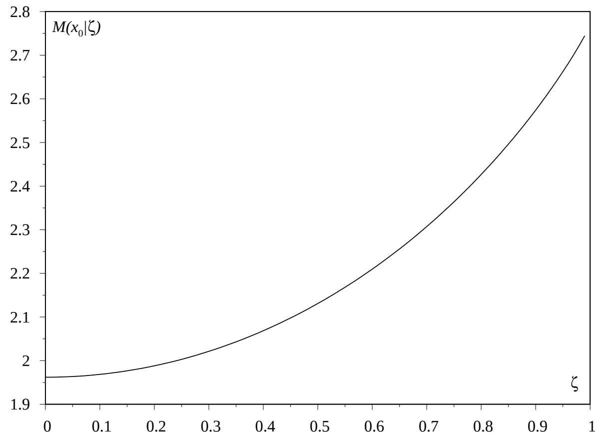

Figure 8: Dependence of dimensionless mass $\mathcal{M}(x_0|\zeta)$ on the parameter ζ.

5. Conclusions

In our work we have shown that the degree of spin-polarization and the influence of finite temperature effects of the electron subsystem can affect the macroscopic characteristics of degenerate dwarf, in particular increase the radius and especially the mass comparing with the standard Chandrasekhar model. The influence is stronger on dwarfs with low masses and is weak for massive ones. We have found the parameters of subsample of massive dwarfs in the model, where both the temperature effects and spin-polarization are taken into account. In the frame of our model we have examined the influence of general relativity effects on the critical mass of degenerate dwarf at which the instability occurs and have shown that it can exceed the Chandrasekhar limit up to 40%, what allows to explain the existence of super-Chandrasekhar white dwarfs.

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