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CORES IN DARK MATTER HALOES WITH ANISOTROPIC OSIPKOV-MERRITT DISTRIBUTION AND MAXIMAL PHASE-SPACE DENSITY

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ABSTRACT. This paper describes the developed model of dark matter cored density profiles. This model was recently proposed by Dmytro Iakubovskyi (Iakubovskyi & Rudakovskyi, in preparation). It has only one extra parameter – the maximal value f_{max} of phase-space distribution function – that turn a cusped Navarro-Frenk-White density profile into a cored one. This paper focuses on the estimation of the influence of velocity anisotropy on the cored density profile by using the Osipkov-Merritt model. The density profiles of the typical dwarf-spheroidal galaxy for different masses of fermionic dark matter particle and different anisotropy parameters r_a was calculated. It was obtained that the influence of velocity anisotropy on the cored density profile is small.

Keywords: Dark matter, core, Navarro-Frenk-White profile, Eddington transformation, Osipkov-Merritt model, dwarf spheroidal galaxy.

1. Introduction

The nature of dark matter particles is one of the most important questions of astroparticle physics and cosmology. The dark matter particle candidates could be classified into two groups: warm dark matter (WDM) particles with relativistic initial velocities (Bisnovatyi-Kogan & Novikov, 1980) and cold dark matter (CDM) with non-relativistic initial velocities (Blumenthal et al, 1984). The standard cosmology ΛCDM model can predict well the large-scale structure of the Universe. But there is no solid evidence that the ΛCDM predictions are successful on the small scales.

The recent numerical simulations of cold dark matter suggest the density distribution of dark matter in the haloes can be described by the Navarro-Frenk-White (NFW) profile (Navarro, Frenk, White, 1996; Navarro, Frenk, White, 1997):

$$\rho_{NFW}(r) = \frac{\rho_0}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2} \tag{1}$$

where r_s and ρ_0 are the parameters of the halo. The parameters concentration C_{200} and halo mass M_{200} are rather used. The M_{200} is the mass of sphere with average density 200 times larger than the critical density of the Universe. The radius of this sphere r_{200} is connected with r_s as the $r_{200} = C_{200} r_s$.

The Navarro-Frenk-White profile has a singularity (cusp) at $r = 0$. But the observations (see, e.g. de Block, 2010) prefer the much flatter shape of density profile (core with constant density near the center of the halo). The cores could be naturally produced by the feedback of supernovae (Navarro, Eke, Frenk, 1996; Pontzen & Governato, 2012), in the fermionic (see, e.g., Ruffini & Stella, 1983) or self-interacting dark matter paradigm (e.g. Kamada et al., 2017). In this paper, we focus on the fermionic particles as DM candidates.

According to the Liouville theorem, the phase-space density of fermionic warm dark matter cannot be larger than some maximal value f_{max}. This fact leads to the limits on the mass of DM particle (Tremaine & Gunn, 1979; Boyarsky et al., 2009). The previous analytical models of dark matter haloes as self-gravitating fermionic gas (Ruffini & Stella, 1983; Bilic & Viollier, 1997; Merafina & Alberti, 2014; Chavanis, Lemou, Mehats, 2015; Domcke & Urbano, 2015; Vega & Sanchez, 2016; Arguelles et al., 2016; Di Paolo, Nesti, Villante, 2017) require non-trivial assumptions about the temperature of dark matter and their distribution function. In this paper, we propose the method of calculating cored density distributions without such disadvantages. We used such cosmological constants from the*Planck* satellite : $\Omega_0 = 0.307$, $h = 0.678$, $\Omega_\Lambda = 0.693$, $\Omega_b = 0.0483$ (Planck collaboration, 2016).

This paper is structured as follows. Sec. 2.1 briefly describes the model of cored density profiles proposed by Dmytro Iakubovskyi (Iakubovskyi & Rudakovskyi, in preparation). Sec. 2.2 focuses on the developed model, which includes velocity anisotropy according to Osipkov-Merritt model. Finally, Sec 3. presents short discussion and conclusion.

2. Methods

2.1. Isotropic halo density profiles

If the velocity distribution of the dark matter particles is isotropic (the distribution function depends only on the energy of the particle), the phase-space density distribution is connected with density profile by the Eddington transformation (Binney & Treemaine, 2008) :

$$f(E) = \frac{1}{\pi^2\sqrt{8}}\frac{\mathrm{d}}{\mathrm{d}E}\int_E^0 \frac{\mathrm{d}\rho}{\mathrm{d}\Phi}\frac{d\Phi}{\sqrt{E-\Phi}}. \quad (2)$$

where E is total energy, $E = \frac{v^2}{2} + \Phi(r)$, v is velocity of the dark matter particle, $\Phi(r)$ is gravitational potential. The gravitational potential is defined as:

$$\Phi(r) = 4\pi G_N \int_0^r \frac{\mathrm{d}x}{x^2}\int_0^x \rho(y)y^2\mathrm{d}y, \quad (3)$$

where G_N is gravitational constant.
The phase-space density corresponding to the NFW profile is infinite at the center of the halo. But the phase-space density of the fermionic gas cannot exceed the maximal phase-space density (Boyarsky et al., 2009):

$$f_{max} = \frac{gm_{FD}^4}{2(2\pi\hbar)^3} \quad (4)$$

where g is internal degrees of freedom of fermions (we assume that $g = 2$), m_{FD} is the mass of the fermion particle.
Hence, following method of obtaining the cored density profile was proposed. This method is based on the assumption that the mass density follows NFW law at large scale and phase-space density distribution function is limited by maximal value f_{max}. The phase-space density distribution function corresponding to staring NFW profile is calculated. Assuming that the DM particle mass is m_{FD}, we truncate obtained distribution function $f(E)$ by the maximal value f_{max}:

$$f_{tNFW}(E) = \begin{cases} f_{NFW}(E), & f(E) < f_{max} \\ f_{max}, & f(E) \geq f_{max} \end{cases} \quad (5)$$

The truncated phase-space density distribution function can be converted to the modified truncated matter density profile:

$$\rho_{tNFW}(r) = 4\pi\int_{\Phi(r)}^0 f_{tNFW}(E)\sqrt{2\,(E-\Phi(r))}\mathrm{d}E \quad (6)$$

New truncated density profile $\rho(r)$ corresponds to the new gravitational potential and new phase-space density distribution function. Hence, the required cored density profile can be obtained iteratively. It was obtained that the 5 iterations are enough for convergence of truncated density profiles.
The dwarf spheroidal galaxies are the objects with highest average phase-space density (Boyarsky et al., 2009). I assume that the density profile of dwarf spheroidal galaxy on large scales is similar to the NFW profile with parameters $M_{200} = 10^8 M_\odot$ and $C_{200} = 30$. I choose the possible values of DM particle mass as $m_{FD} = 250, 500, 1000, 2000$ eV and ∞ (this case corresponds to the NFW profile at all radii). The obtained density profiles are depicted on Figure 1.

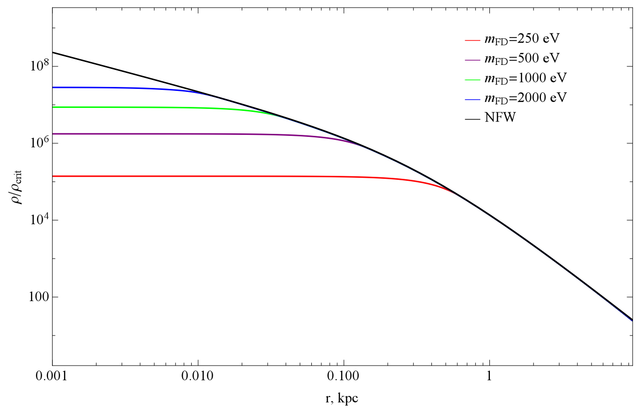

Figure 1: Cored halo density profiles of typical dwarf spheroidal galaxy for fermionic dark matter particles with masses $m_{FD} = 250, 500, 1000, 2000$ eV. Velocity distribution is isotropic. NFW profile corresponds to the $m_{FD} = \infty$, its parameters are $M_{200} = 10^8 M_\odot$ and $C_{200} = 30$ respectively.

2.2. Cored density profile in Osipkov-Merritt model

The anisotropy of velocities in the halo is described by the parameter $\beta(r) = 1 - \frac{\sigma_t^2(r)}{\sigma_r^2(r)}$, where σ_t and σ_r are tangential and radial velocity dispersions. The density profiles in previous sections were obtained in the assumption that $\beta = 0$ ($\sigma_r = \sigma_t$). In this section, we estimate the influence of anisotropy on cored profile by using the Osipkov-Merritt model (Osipkov, 1979; Merritt, 1985; Binney & Tremaine, 2008). The Osipkov-Merritt model is based on the assumption that the phase-space density distribution function depends on the isolated integral of motion $Q = E + \frac{L^2}{2r_a^2}$, where E is the energy, L is angular momentum, r_a is anizotropy radius (parameter of the halo). The anisotropy parameter in this model is $\beta = 1 - \frac{\sigma_t^2}{\sigma_r^2} = 1 - \frac{1}{1+\frac{r^2}{r_a^2}}$. For $r \ll r_a$ the velocity distribution is isotropic, and anisotropic for large radii. The case $r_a \to \infty$ corresponds to the isotropic velocity distribution.

Denoting the $\rho_Q(r) = \left(1 + \frac{r^2}{r_a^2}\right)\rho(r)$, the Eddington's transformation changes to:

$$f(Q) = \frac{1}{\pi^2\sqrt{8}}\frac{\mathrm{d}}{\mathrm{d}Q}\int_Q^0 \frac{\mathrm{d}\rho_Q}{\mathrm{d}\Phi}\frac{\mathrm{d}\Phi}{\sqrt{Q-\Phi}}. \quad (7)$$

The phase-space density distribution function $f(Q)$ is truncated by f_{max}:

$$f_{tNFW}(Q) = \begin{cases} f(Q), & f(Q) < f_{max} \\ f_{max}, & f(Q) \geq f_{max} \end{cases} \quad (8)$$

Than truncated density profile is calculated as follows:

$$\rho_{tNFW}(r) = \frac{4\pi}{\left(1 + \frac{r^2}{r_a^2}\right)}\int_{\Phi(r)}^0 f_{tNFW}(Q)\sqrt{2\left(Q - \Phi(r)\right)}\mathrm{d}Q \quad (9)$$

In this paper 5 iterations was used in iterative process (similarly to previous section). The obtained results for typical dwarf spheroidal galaxy, $m_{FD} = 500$ eV and $r_a = 0.25, 0.5, 1$ kpc is depicted on Fig. 2.

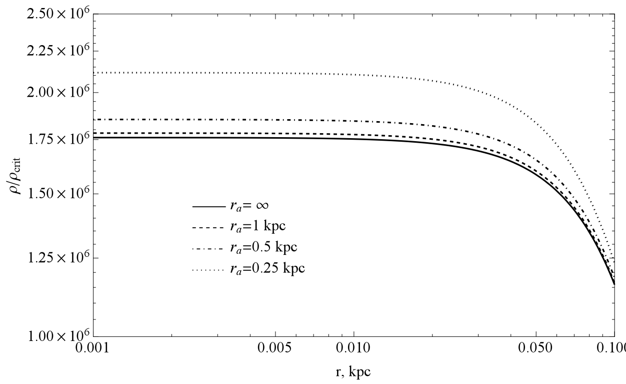

Figure 2: Cored density profiles of typical dwarf spheroidal galaxy with anisotropy radiuses $r_a = 0.25, 0.5, 1, \infty$ kpc and $m_{FD} = 500$ eV. $r_a = \infty$ corresponds to the isotropic cored halo.

3. Discussion and conclusion

This paper focuses on the development of the iterative method of calculation of the cored halo density profile for fermionic dark matter particles (proposed by Dmytro Iakubovskyi; Iakubovskyi & Rudakovskyi, in preparation). According to this method, the phase-space density distribution $f(E)$ (corresponding to the NFW profile) was calculated by using the Eddington transformation. Then $f(E)$ was truncated phase-space density by some maximal value f_{max} and recalculated corresponding truncated mass density ρ_{tNFW}. This procedure was iterated until ρ_{tNFW} converges. On large scales ρ_{tNFW} is well described by NFW profile. But near the center of the halo, the density is flattening. This core corresponds to the degenerate fermionic dark matter gas, which phase-space density cannot exceed some maximal values f_{max}. The obtained shape of ρ_{tNFW} is analogous to the results of the simulations in (Shao et al., 2013; Maccio et al., 2013).

The cored density profile is characterized by core radius r_c. In this paper the core radius is defined as $\rho_{tNFW}(r_c) = \frac{1}{4}\rho_{tNFW}(0)$. In this paper the density profile of typical dwarf spheroidal galaxy is assumed on large scales as described by NFW with parameters $M_200 = 10^8 M_\odot$, $C_{200} = 30$. It was found for this halo that $r_c \simeq 0.03$ kpc for $m_{FD} = 2000$ eV, $r_c \simeq 0.2$ kpc for $m_{FD} = 500$ eV, $r_c \simeq 0.7$ kpc for $m_{FD} = 250$ eV. Obtained density profiles showed the decreasing density of core $\rho_{tNFW}(0)$ with increasing the mass of DM particle. The difference in M_{200} of initial NFW profile and M_{200} of obtained cored density profile for dwarf spheroidal galaxy does not exceed 10% .

The influence of velocity anisotropy on the cored density profile was estimated by using the Osipkov-Merritt (OM) model. OM model assumes that the distribution function depends on isolated integral of motion $Q = E + \frac{L^2}{2r_a^2}$ instead of E. The natural assumption is that the smallest value of anisotropy parameter r_a must be comparable with the core radius of isotropic halo r_c. This assumption is based on the fact that central parts of haloes seem to be isotropic (Boyarsky et al., 2009). In this paper I focus on the $m_{FD} = 500$ eV, hence the minimal $r_a = 0.25$ kpc was chosen. It was obtained that the influence of anisotropy on a radius of a core is negligible for all tested r_a. The density in the core is increased maximum by 20 % in the presence of velocity anisotropy. For $r_a \geq 1$ kpc the difference between the anisotropic and isotropic density profiles is negligible.

Developed model of cored density profile combined with the models of baryonic feedback processes and observations of dwarf spheroidal galaxies can be used for constraining mass of fermionic dark matter particle candidate.

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