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LIMIT VALUES AS AN UNIVERSAL METHOD OF DESCRIPTION OF PHYSICAL REALITY

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ABSTRACT. There are two main forms of cognition of Nature: empiric knowledge obtained from the experience, and theory as system of ideas and principles. Each of the two forms uses its own methods. While empirics or phenomenology is based on experiments, theory mostly deals with axiomatic approach. Every axiomatics starts from the principal question: what statements should be chosen as axioms? The present work uses the existence of limit values as the initial axiom. It is well known that the statement about the existence of minimum quantum of action $\hbar$ is sufficient to build all the Quantum Mechanics, likewise maximum velocity value c—for the Special Relativity. Similar approach can be realized in General Relativity as well, which can be built on the postulated existence of limit (maximum) power

$$\eta = \frac{c^5}{4G}.$$

It seems natural in context of this axiomatics to transit from the traditional Planck's units to the modified ones, i.e. from the set ($\hbar$, c, G) to ($\hbar$, c, η), with the latter containing exclusively limit values. The approach considered in the present paper opens new exciting possibilities for interpretation of the known results and obtaining the new ones.

Keywords: axiomatic approach, limit values, fundamental constants.

1. Introduction

The traditional approach to describe the reality was based on physical laws and seemed unshakable, but today it is changing before our very eyes. The new concept "it from bit" (Wheeler, 1986; Lloyd, 2005) step by step conquers its place in collective consciousness of the "physical community" (holographic principle, black hole thermodynamics, informational paradox), and number of its supporters continuously grows. Structure of the new paradigm can be briefly described in the following way.

States of a physical system should be treated as purely informational states. Space-time, where all physical processes are played out in the habitual picture of the world, now is "just" an object for realization of the informational states. The information (the bits) is now the only real thing.

Such a radical revision of the reality nature causes understandable prejudices, especially in view of undeniable achievements of the traditional physics. And the first question to rise is: why Nature uses two dominant approaches instead of one? It is possible that the two alternative approaches can contradict each other. Time reversibility of mechanics and time arrow in thermodynamics represent a well known example of such a contradiction. Hopefully Nature is sufficiently perceptive to avoid contradictions of that kind. It has apparently foreseen that all the adequate approaches to its description are somehow, yet mysteriously for us, linked.

In our opinion, the so-called limit values play an important role in search for those links. The statement about the existence of limit values can be used as a basis for physical axiomatics. It is well known that the Quantum Mechanics can be built basing on the existence of minimum quantum of action $\hbar$, Special Relativity—of maximum velocity c. Relatively recently it became clear that the analogous approach can be realized in general Relativity as well (Gibbons, 2002; Schiller, 2006), which can be built on postulate about existence of maximum force

$$F_{max} = \frac{c^4}{4G}.$$

The limit values are actual for all physical systems regardless their nature, and for every observer. The particular value of the limit is of less importance than the very fact of its existence. One should distinguish between two types of the limit values: the "fundamental" ones and all the others. The limit value is called fundamental if it cannot be deduced from the existing theories, and its existence can be used as a basis for future theories. A classical example: finite value of propagation velocity for arbitrary signal in vacuum generates Lorentz transformations and consequently the Special Relativity.

Only finite limit values make interest. Appearance of singularities in a theory is commonly considered as a first signal of the fact that the theory has gone beyond its applicability limits and needs modernization. The latter implies taking into account the previously neglected effects, which would allow to make the theory free of the singularities.

An alternative point of view was formulated by Penrose as the "cosmic censorship principle": Nature always hides a naked singularity (Penrose, 1973). The space-time singularities appear in such places which are hidden from observers, like inner parts of black holes. In other words, the super-limit values are screened from us by the horizons, and the very limit is reached exclusively at the horizon. The present work is aimed to generalize the cosmic censorship principle to the level of the physical censorship one, having shown that a large number of fundamental limitations in both micro- and macro-physics is imposed by existence of the limit values. Treatment of those limitations in terms of the limit values opens new and interesting possibilities for axiomatic formulation of physics, which can be realized postulating existence of certain set of limit values.

2. The Maximum Force Principle

The role of the two fundamental values – the light speed c and Planck's constant $\hbar$ – is well known. We will now focus on the third fundamental constant— the limit force. The maximum force principle was first formulated in the paper of Gibbons (Gibbons, 2002): *I suggest that classical General Relativity in four space-time dimensions incorporates a Principal of Maximal Tension and give arguments to show that the value of the maximum tension is*

$$F_{max} = \frac{c^4}{4G} \approx 3.25 \times 10^{43} N. \quad (1)$$

The limit does not depend on the force nature and equally holds for gravitational, electromagnetic, nuclear, and all other forces. The statement about existence of maximum power is absolutely equivalent to the latter:

$$P_{max} = \frac{c^5}{4G} \approx 9.07 \times 10^{51} W. \quad (2)$$

Both quantities are components of the 4-vector

$$F^\lambda = \frac{dp^\lambda}{dt}.$$

The multiplier 1/4 does not play a principal role. Therefore, further, where it does not lead to confusion, we will omit numerical multipliers of the order of unity assuming that the approximate equality $A \approx B$ corresponds to the relation $\log A \approx \log B$.

The limit force and the limit power are invariants: if follows from invariance of the quantities c and G. Time dependence is not however excluded in general. The force limit takes place for every component of the 3−force, as well as for its absolute value.

The limit power has a trivial physical interpretation. Let us consider the power released in "annihilation" of a black hole of mass M. Minimum time required for realization of this process equals to the time interval needed for light signal to travel the distance equal to its "gravity radius"

$$t = \frac{2R_g}{c} = \frac{4MG}{c^3}$$

$$P = \frac{Mc^2}{4MG/c^3} = \frac{c^5}{4G}, \quad (3)$$

which exactly coincides with the above introduced limit power.

Here is another example to clarify the mechanism of the occurrence of the maximum force. In the Newtonian mechanics $F = dp/dt$, therefore

$$F_{max} = \frac{(\Delta p)_{max}}{(\Delta t)_{min}} \approx \frac{mc}{t_{Pl}} = \frac{mc^2}{l_{Pl}}. \quad (4)$$

At first sight one may expect that unlimited growth of the mass will give rise to arbitrarily great force. However, this is not true, and the limitation is bound up with the appearance of horizon at the increase of the mass on a fixed scale of length l_{Pl}. Indeed, omitting the numerical multipliers $O(1)$, we find the mass with the gravitational radius equal to the Planck length:

$$m \approx \frac{l_{Pl}c^2}{G} = \sqrt{\frac{\hbar G}{c^3}}\frac{c^2}{G} = \sqrt{\frac{\hbar c}{G}} = m_{Pl}. \quad (5)$$

Consequently, the maximum mass which can be used in (3) for preventing the appearance of the horizon is the Planck mass, so

$$F_{max} \approx \frac{m_{Pl}c^2}{l_{Pl}} = \frac{c^4}{G}. \quad (6)$$

The result (6) can be obtained in the form of the combination of the Planck units with the dimension of force:

$$F_{Pl} = m_{Pl}\frac{l_{Pl}}{t^2_{Pl}} = \sqrt{\frac{\hbar c}{G}}\sqrt{\frac{\hbar G}{c^3}}\frac{c^5}{\hbar G} = \frac{c^4}{G}. \quad (7)$$

Note that for the Planck mass the gravitational radius coincides with the Compton wavelength.

It should be emphasized that all our statements concern solely the case of dimension $D = N + 1 = 4$. It is only in the space of dimension $D = 4$ that the Planck force is independent of $\hbar$:

$$F_{Pl(D)} = \frac{M_{Pl(D)} L_{Pl(D)}}{T^2_{Pl(D)}} = G_D^{-\frac{2}{D-2}} \hbar^{D-4} c^{\frac{D+4}{D-2}}. \quad (8)$$

More strictly the expression for the limit force (limit power) can be obtained in the frames of General Relativity (Schiller, 2005).

3. Modification of Planck's Unit System

Dimensional analysis is a powerful method which makes it possible to obtain results (both qualitative and quantitative) on the basis of general knowledge of the phenomenon under consideration. Dimensional analysis (along with symmetry considerations) is especially significant for construction of initial approaches to description of the systems for which any theory is absent at present. As it is well-known, many astonishing results have been achieved due to dimensional considerations which at first sight seem to be quite simple. There are even such results that are still not obtained in other, more rigorous way. A classical illustration of such a situation is quantum gravity. The latter has practically become a synonym of Planck-scale physics whose description to a considerable extent reduces to endless shuffle of fundamental constants. However, dimensional analysis is not all-powerful, and the results obtained with its help should be interpreted carefully.

Especially significant role in clarification and understanding of the foundations for the future theory of Planck-scale processes belongs to the Planck units. The Planck units represent fundamental physical scales of mass, length and time built by means of the fundamental constants $\hbar, c, G$ (Planck, 1899):

$$\begin{aligned} m_{Pl} &= \sqrt{\frac{\hbar c}{G}} \simeq 2.18 \times 10^{-8} kg, \\ l_{Pl} &= \sqrt{\frac{\hbar G}{c^3}} \simeq 1.6 \times 10^{-35} m, \\ t_{Pl} &= \sqrt{\frac{\hbar G}{c^5}} \simeq 5.39 \times 10^{-44} sec \end{aligned} \quad (9)$$

Planck units represent "natural" physical scales of mass, length and time, constructed from the fundamental constants $\hbar, c, G$. The three constants used to construct the Planck's units have different functional roles. While the first and the second of them represent limit values and lie in the foundations of Quantum Mechanics ($\hbar$) and Special Relativity (c), the Newtonnian constant (G) "just" fixes absolute value of the gravitational forces. It seems natural to make the set of fundamental constants more consistent and more efficient using exclusively limit values to construct the Planck units. In order to that, in addition to the limit values $\hbar$ and c, we introduce the limit power $\eta = P_{max}$, having made the substitution

$$G = \frac{c^5}{\eta}.$$

In other words, using the set $(\hbar, c, \eta)$ instead of $(\hbar, c, G)$, we get the modified system of Planck units (which consists only of the limit values):

$$\begin{aligned} m_{Pl} &= \sqrt{\frac{\hbar c}{G}} \rightarrow m_{Pl} = \sqrt{\frac{\hbar \eta}{c^4}}, \\ l_{Pl} &= \sqrt{\frac{\hbar G}{c^3}} \rightarrow l_{Pl} = \sqrt{\frac{\hbar}{\eta}} c, \\ t_{Pl} &= \sqrt{\frac{\hbar G}{c^5}} \rightarrow t_{Pl} = \sqrt{\frac{\hbar}{\eta}}. \end{aligned} \quad (10)$$

It should be emphasized again that the necessary condition for the existence of event horizon is finiteness of the realized power and of the speed of light. Thereat, as we have already noted, the magnitude of the limit value is less significant than the fact of its existence. It is easily seen that

$$\lim_{c \to \infty} R_g = \lim_{c \to \infty} \frac{2mG}{c^2} = 0;$$

$$\lim_{P_{max} \to \infty} R_g = \lim_{c \to \infty} \frac{2mc^3}{P_{max}} = 0. \quad (11)$$

$$(12)$$

In other words, at $\eta \to \infty$ or $c \to \infty$ the concept of gravitational radius and, consequently, the event horizon, becomes meaningless.

While considering the maximum force (maximum power) as a fundamental constant it is natural to use it instead of the gravitational constant. For instance, Newton's law of universal gravitation acquires the form

$$F = G\frac{mM}{R^2} = \frac{c^4 mM}{4F_{max} R^2} = \frac{1}{F_{max}} \frac{mc^2 \cdot Mc^2}{R^2} \quad (13)$$

Here the relation between the value of gravitational interaction and the maximum force becomes more transparent: a gigantic maximum force gives rise to a weak gravitational interaction. Naturally, if one chooses the gravitational constant G for the initial fundamental constant, the opposite statement will be true as well.

The choice of the maximum power as a new fundamental constant leads to the Planck scales which preserve their previous numerical values. However, such a changeover opens up interesting opportunities for interpretation of the estimates made using the modified Planck units, as well as for the obtaining the new results. Below we will give a number of examples.

4. Space-Time Foam

If the space is subject to quantum fluctuations, then the fluctuations should manifest in form of the uncertainties in measurements of different type (Ng & van Dam, 1994). Measurement of length is an important class of measurements. One can measure length of an interval measuring time of registration of the reflected signal. However the quantum fluctuations will generate uncertainty δl of the measured distance. Wigner (1957) showed that

$$\delta l^2 \geq \frac{\hbar l}{mc}. \tag{14}$$

Here m is the clock's mass. It may seem that increasing mass of the clock infinitely, one can eliminate influence of the quantum fluctuations. However, possibility to increase the clock's mass is strictly limited. The clock's characteristic size d is evidently limited by the very experiment's validity condition ($d \leq \delta l$), and on the other hand the clock's size must exceed its Schwarzschild radius $d > Gm/c^2$, preventing the transformation of the clock into a black hole, because otherwise the clock readings would be unavailable to us. It then follows that

$$\delta l \geq \frac{Gm}{c^2}. \tag{15}$$

Combining (14) and (15), one obtains (Karolyhazy, 1966)

$$\delta l \geq (ll_{Pl}^2)^{1/3} = l_{Pl}\left(\frac{l}{l_{Pl}}\right)^{1/3}, \quad l_{Pl} \equiv \sqrt{\frac{\hbar}{\eta}}c. \tag{16}$$

Similar relations can be obtained for measuring time intervals (Ng, 2001; 2002),

$$(\delta t)^2 \geq \frac{\hbar t}{mc^2}, \quad \delta t \geq \frac{Gm}{c^3} \tag{17}$$

where t is the measured time interval. By combining these two expressions we find

$$\delta t \geq \left(tt_{Pl}^2\right)^{1/3} \tag{18}$$

Relation (18) connects the minimum uncertainty during measurement of time with the measured time interval. The absolute value of uncertainty $\delta t \sim t^{1/3}$ rises, whereas its relative value $\delta t/t \propto t^{-2/3}$ diminishes. Now rewrite relations (16) and (18) in the form

$$\delta l \geq \left(\frac{l}{c}\frac{\hbar}{\eta}\right)^{1/3} c; \tag{19}$$

$$\delta t \geq \left(t\frac{\hbar}{\eta}\right)^{1/3} \tag{20}$$

which clearly shows that the existence condition for the minimum uncertainty during measurement of distance (time) is equivalent to the existence condition for the limit power which, in its turn, is dictated by the existence of the horizon.

To avoid confusion, we emphasize that in the first and in the second case it is not about the accuracy of the particular design of "ruler" or clock, and the universal limitations on the accuracy of the measurement of length and time, which are based on fundamental physical laws.

We should point out that the result (19) is closely linked to the holographic principle ('t Hooft, 1994; Susskind, 1995), according to which all the information contained in some region of space can be "recorded" (represented) on the boundary of the region.

Let us imagine that some volume l^3 is divided on parts (say, cubes) of the smallest size that is allowed by physical laws. It seems natural to assign one degree of freedom to each such elementary volume (recall dimensionless cell of phase volume of a quantum system $dpdq/(2\pi\hbar)^{3N}$). If the minimum uncertainty in measurement of distance l equals to δl, then the elementary volume component has volume $(\delta l)^3$ and number of degrees of freedom in the system equals $(l/\delta l)^3$. According to the holographic principle

$$(l/\delta l)^3 \leq \frac{l^2}{l_{Pl}^2}, \tag{21}$$

which immediately returns us to the result (19).

It is important to note that in derivation of (21) we used the holographic principle to find the expression for the minimum uncertainty δl. As we have seen above, existence of this fundamental characteristic directly follows from the physical censorship principle (the maximum force principle) and its value can be obtained without application of the holographic principle. Therefore with the same degree of certainty we can assert that the holographic principle represents a consequence of the quantum fluctuations of the space-time (J. Ng, 2003).

5. Maximum Acceleration

The existence condition for the traditional space-time in the presence of vacuum polarization (virtual processes of production and annihilation of pairs caused by quantum fluctuations) leads to limitation of proper acceleration relatively to the vacuum, or, in other words, to the occurrence of the maximum acceleration (Caianiello, 1981; Brandt, 1989; Papini, 2003; Wood, 1989).

The proper acceleration of the particle a in curved space-time is the scalar defined by the relation

$$a^2 = -c^4 g_{\mu\nu}\frac{Dv^\mu}{ds}\frac{Dv^\nu}{ds} \tag{22}$$

where $g_{\mu\nu}$ is the metric tensor, $v^\mu \equiv dx^\mu/ds$, the dimensionless four-velocity of the particle, D/ds is the covariant derivative with respect to the line element on the world line of the particle,

$$\frac{Dv^{\mu}}{ds} \equiv \frac{dv^{\mu}}{ds} + \Gamma^{\mu}_{\alpha\beta} v^{\alpha} v^{\beta} \tag{23}$$

Here $\Gamma^{\mu}_{\alpha\beta}$ are the affine connections (the Christoffel symbols) of space-time with the metric $g_{\mu\nu}$, $ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu}$, the linear element of this space-time.

From the energy-time uncertainty principle it follows that the lifetime of the virtual pair particle-antiparticle (with the particle mass m) generated due to vacuum fluctuations, is $\approx \hbar/2mc^2$, whereas the distance covered during this time is $\approx \hbar/2mc$ (the Compton wavelength of the particles). If a virtual particle acquires the energy equal to its rest mass, it will be transformed into a real particle. When considering the rest system of a particle which is, generally speaking, non-inertial, we find that it undergoes the inertial force $F_{in} = |ma|$, where a is the proper particle acceleration. The work executed by the inertial force during the particle lifetime

$$A = ma \times \frac{\hbar}{2mc}.$$

If $A = mc^2$, then there arises acceleration

$$a = \frac{2mc^3}{\hbar} \tag{24}$$

At this acceleration, particles of the mass m will be copiously produced from the vacuum. The growth of acceleration will lead to the rise of the mass of the produced particles. What critical consequences may arise at unlimited growth of acceleration? If the value of acceleration is high enough, the produced particles can be transformed into black holes. This will occur in the case when the Compton wavelength of a particle (particle "size") $\hbar/mc$ is less than its Schwarzschild radius $2Gm/c^2$,

$$\hbar/mc < \frac{2Gm}{c^2} \tag{25}$$

From here it follows that the threshold for black hole formation is a mass of the order of the Planck mass $(\hbar\eta)^{1/2}/c^2$. By substituting $m = m_{Pl}$ into (1) we find

$$a_0 \approx \sqrt{\frac{\eta}{\hbar}}\, c \tag{26}$$

(as before, we omit the multipliers of the order of unity). At such an acceleration, production of black holes with the Planck mass due to vacuum polarization will result in breakdown of the traditional knowledge of the structure of space-time, and the acceleration concept itself will lose its conventional sense. Therefore, the value a_0 should be considered the maximum proper acceleration relatively to the vacuum. Note that the presence of the maximum acceleration leads to the formation of a horizon even in SR. In fact, from the viewpoint of SR, the length l of an object moving with the acceleration a is limited by the relation

$$l \leq \frac{c^2}{2a}.$$

On the other hand, it cannot be less than $l \geq l_{Pl} = \sqrt{\hbar/\eta}c$. When using this inequality for acceleration one obtains

$$a \leq c\sqrt{\frac{\eta}{\hbar}}$$

. As is seen, the maximum acceleration corresponds to the fundamental acceleration in the Planck system of units, and is a simple combination of the three limit values $\hbar, c, \eta$. The necessary condition for its existence is finiteness of all the three limit values : at $c \to \infty$, $\hbar \to 0$ or $\eta \to \infty$ the maximum acceleration is absent.

The presence of the maximum proper acceleration a_0 (33) automatically leads to the existence of the minimum radius of curvature $R_{\min}$ of the particle world lines. The radius of curvature of the world line is $R = c^2/a$ (since the centripetal acceleration during motion along the circle of the radius R is $a = v^2/R$). Therefore, the minimum radius of curvature has the form

$$R_{\min} = \frac{c^2}{a_0} \approx \left(\frac{\hbar G}{c^3}\right)^{1/2} = c\left(\frac{\hbar}{\eta}\right)^{1/2} \tag{27}$$

Again, we clearly see the key role of the horizon which produces the limit power and, as a consequence, the maximum proper acceleration and the minimum radius of curvature of the world line.

6. An Ideal Quantum Clock and Principle of Maximum Force

Achievement of required accuracy in any quantum measurement inevitably imposes certain limitations on characteristics of the device designed to perform it. All possible methods to measure the time always involve observation of some periodical physical process. As an example (following (Burderi, 2016)), consider a quantum clock based on observation of radioactive disintegration described by the following equation

$$\frac{dN}{dt} = -\lambda N \tag{28}$$

where $N(t)$ is the current number of radioactive particles in the sample. Average number of the decayed particles during the time interval $\Delta t \ll \lambda^{-1}$ is $\Delta N = \lambda N \Delta t$. It enables us to measure the time intervals calculating number of the decaying particles

$$\Delta t = \frac{\Delta N}{\lambda N} \tag{29}$$

The relative error of such a method of time measurement $\varepsilon = (\lambda N \Delta t)^{-1/2} = 1/\sqrt{\Delta N} \leq 1$. At first glance, it seems that increasing size of the quantum clock (the number N), we would gain unlimited improvement in accuracy of the time interval measurement. However, such a process is limited by the following condition: the rise of the clock mass must not lead to transformation of the clock into a black hole (i.e. to occurrence of horizon). Let us analyze the quantitative limitations which may be caused by this condition. By using the uncertainty principle $\Delta E \Delta t \geq \hbar/2$ we can transform (29) into the inequality

$$\Delta t \geq \frac{\hbar}{2\varepsilon^2 c^2}\frac{1}{M} \tag{30}$$

where $M = Nm_p$ (with m_p corresponding to the mass of one particle) is the clock mass. If the clock radius R (the clock is assumed to be spherical) becomes less than the gravitational radius R_g, it will be impossible to use the clock for time measurements. The condition $R > R_g$ is transformed into

$$\frac{1}{M} > \frac{2G}{c^2 R} \tag{31}$$

When substituting (31) into (30) we obtain

$$\Delta t R > \frac{1}{\varepsilon^2}\frac{G}{c^4}\hbar \tag{32}$$

Treating R as uncertainty Δr in position of the physical object (the clock), which is the basis for the time measurement process, and taking into account that $\varepsilon \leq 1$, one finally obtains (Bolotin, 2016)

$$\Delta t \Delta r > \frac{G}{c^4}\hbar \tag{33}$$

The obtained inequality limits the possibility to determine the time and space coordinates of events to an arbitrary precision.

Let us analyze expression (44) using the notion of the limit force (Bolotin, 2016). For this purpose present it in the form

$$\Delta t \Delta r > \frac{1}{F_{\max}}\hbar \tag{34}$$

At the fixed Planck constant $\hbar$, it is only the limit force $F_{\max}$ defines the limitation imposed on the quantum clock size. If such a force is absent in the theory, i.e. $F_{\max} = \infty$, then $R_g \to 0$, and limitation for the quantum clock size is absent too. The main cause of the discussed limitation is the requirement $R > R_g$ equivalent to the condition preventing the formation of horizon. Therefore, the occurrence of the force $F_{\max}$ in relation (34) which can be achieved only at the horizon seems absolutely natural.

The structure of relation (34) does not contain any information concerning the process which has been the base for construction of the clock. This suggests the idea that this relation may be obtained from general considerations. To prove this statement let us use the uncertainty relation

$$\Delta x_{\min} \Delta p_{\max} \geq \frac{\hbar}{2} \tag{35}$$

Since

$$F_{\max} = \frac{\Delta p_{\max}}{\Delta t_{\min}},$$

we immediately obtain that the minimum size $\Delta x_{\min}$ of the clock necessary for measurement of the time intervals $\Delta t_{\min}$ obeys the limitation

$$\Delta x_{\min} \Delta t_{\min} \geq \frac{\hbar}{F_{\max}} = \frac{\hbar c}{\eta} \tag{36}$$

in complete correspondence with (45). This is just the relation that describes the structure of space-time foam! A simple form of relation (47) points to the fact that the limit values $\hbar, c, \eta$ have a fundamental character.

Certainly, the earlier obtained restrictions for the limits of measurability of distance and time (20) are in accord with relations (47). In fact, multiplication of the uncertainties (20) gives

$$\delta l \cdot \delta t \geq \left(\frac{l\hbar}{c\eta}\right)^{1/3} c \left(\frac{t\hbar}{\eta}\right)^{1/3} = (l \cdot t)^{1/3} \left(\frac{\hbar c}{\eta}\right)^{2/3} \tag{37}$$

Suppose that we are to measure the minimum scales of length and time, i.e. $l = \delta l$ and $t = \delta t$. In such a case (37) will reproduce relation (36).

7. Limit Relations in Information Theory

Limit values $\hbar, c, \eta$ control (or limit) rates of all physical processes and in particular information transmission rate. Exact value of this quantity goes far beyond all purely technological applications. Level of human society development to great extent is determined by information transmission and processing speed. In order to find the limit working speed of a computing devise one should take into account three aspects:

1. Uncertainty principle
2. Finite velocity of signal transmission
3. Necessity to prevent formation of a black hole (existence of the horizon)

In the considered example all the three limit values $(\hbar, c, \eta)$ work together leading to the following inequalities:

$$\Delta E \Delta t \geq \hbar, \quad \Delta t \geq L/c, \quad L > r_g = \frac{2MG}{c^2}. \tag{38}$$

It results in the following expression for the limit working speed v of arbitrary computing device (Gorelik, 2009):

$$v = \sqrt{\frac{c^5}{G\hbar}} = \sqrt{\frac{\eta}{\hbar}} = t_{Pl}^{-1}. \qquad (39)$$

The information processing speed in arbitrary computing device is bounded by limit concentration of energy inside the device. The limit power η gives quantitative measure of this bound. At $\eta \to \infty$ (limitations are absent) the computing device could work with arbitrarily high performance.

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