

THE POINT OBJECT'S IMAGE FORMING RAY MODEL

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ABSTRACT. Light wave front's distortions are commonly explained by atmosphere turbulence. But Anler equation solution's analysis shows that distributed homogeneous medium even in the absence of turbulence makes nonlinear transformation of coordinates as well as ray's entry angles to the telescope's optical system (rays' trajectory is a screw-like line).

Key words: Atmosphere, model, parameters.

The atmosphere light propagation in distributed inhomogeneous optical medium is distributed by Au-ler's differential equations (Mikaeljian, 1990). The rays trajectories depened upon medium perts' types when going through them on the way to telescope's image plane. Optical mediums are subdivided on the type of refraction coefficient's $(RC)N[X, Y, Z]$ symmetry into:

1. homogeneous (classical optics) - $N[X, Y, Z] = \text{const}$;

2. distributed isotropic - $N[X, Y, Z] = N[Z]$. In such atmosphere RC increases in ray's direction according to density growing.

3. Anisotropic- $N[X, Y, Z] = N[X, Y]$. Anisotropy is a consequence of local RC disturbances along axes X, Y . Suppose that the telescope axis is Z one; the plane XY coincides with the moving image plane. Au-ler's equation for distributed isotropic medium describes the ray trajectory projection to XZ or YZ plane. For the first case:

$$(X''/1 + (X')) + (1/N[Z]) \cdot (dN[Z]/dZ) \cdot X' = 0; \quad (1).$$

Substitute to (1) function defining $N[Z]$ dependence on medium parameters. Designate angle formed by the telescope axis and the zenith as F and power function M as $M\{s\}$. It is known (Zverev, 1975), that

$$N[Z] = (1+C \cdot P[H])\{-1/2\} = P \cdot (kT/gm)\{-gm/kC\} \cdot e\{(-gm/Bk) \cdot H\}; \quad (2).$$

where $C=\text{const}$; $P[H]$ -medium density; $B = dP[H]/dH$; P, h, T -medium density, height and temperature recordingly at the layer's border. Formula (2) was obtained under assumption that $T[H] = T + B \cdot H$. Taking into account that $H = Z \cdot \cos F$, substitute RC and its derivative to (1), and separating variables, integrate result:

$$dX/dZ = -tg[(\pi/r[F, x])[Z + (g \cos F)(1 - e\{-h +$$

$$S[n] \cos F\} \cdot e\{az \cos F\}]] - Q(x)]; \quad (3).$$

Here $a = gm/kT$; $A = (1/a)\{-gm/kC\}$; $r[F, x]$ - ray's oscillation period in XZ plane; $Q(x)$ - ray's entry angle; $q = C \cdot P \cdot A/a$; $S[n]$ - ray's trajectory length along Z for n previous periods while.

In order to express depedence dY/dZ upon medium parameters designate in (3)

$$dX/dZ = -tg[(\pi/r[F, x])Fi(x, z) - Q(x)]; \quad (3a).$$

$$dY/dZ = -tg[(\pi/r[F, y])Fi(y, z) - Q(y)]$$

and taking into account, that $Q(y) = (\pi/2) - Q(x)$, receive:

$$dY/dZ = ctg[(\pi/r[F, y]) \cdot Fi(F, y) + Q(x)]; \quad (3b).$$

Oscillation periods $r[F, x], r[F, y]$ at XZ and YZ planes correspondingly are found from:

$$(\pi/r[F, x])Fi(x, z) - Q(x) = \pi; \quad (4).$$

$$(\pi/r[F, y])Fi(y, z) + Q(x) = \pi; \quad (4a).$$

$$1 + r[F, x] + (q/\cos F)(1 - e\{-h + S(n0 \cos F)\} \cdot e\{ar[F, x] \cos F\}) + (Q(x)r[F, x])/\pi = 0; \quad (5).$$

Expression (5) represents (4) in detailed form. Let's formulate atmosphere optical characteristics followed from (3), (4) and (4a):

1. Ray's propagation vector rotates when coordinate Z changing and makes one spiral spire for two $r[F, x]$ periods;
2. At the end of every period ray's propagation angle equals to the angle of its entry to the medium. The ray is parallel to Z axis in trajectory points, where phase equals to $\pi/2$;
3. The oscillation frequency increases in successive periods, when decreasing temperature factor $B, Q(x)$ and $Q(y)$ absolute values;
4. Ray's parallelism maintains at the every trajectory point;
5. Medium entry angles equality for axis Z points' rays causes the angles equality when propagating within the medium;
6. Two axis Z points' rays with entry angles $Q(x)$ and $\pi/2 + Q(x)$ correspondingly have equal frequencies and $\pi/2$ phase shift when propagating within the

medium. Rays are meeting or diverging ones subject to phase;

7. Rays cutting image plane along the circle with $(0, 0, Z)$ center are homocentric ones, that is tangents to their trajectories cut one point at Z axis.

The above permits to assert that atmosphere possesses plate characteristics (*it.4*) as well as convex or concave sphere ones (*it.6*). To the specific optical characteristics concern: spiral-like trajectory; conservation of the entry angle's value in whole number of periods; homocentricity of one radius's rays family (*it.2*). Any distortion may be realised as simulation model if angles and rays' entry coordinates to the telescope's objective are known. The entry angles to the objective are determined by residual values of variable Z (deduction by whole values of periods) and expressions (3), (3b). While $z < r[F, x]$ ray's entry coordinates $X[Z], Y[Z]$ are determined by formulas:

$$X[z] = X[0] + U \cdot \ln(\cos Q(x) / \cos[(\pi/r[F, x])[z + q/\cos F(1 - B \cdot e\{az \cos F\})] - Q(x)]); \quad (6a)$$

$$Y[z] = Y[0] + U \cdot \ln(\sin Q(x) / \sin[(\pi/r[F, y])[z + q/\cos F(1 - B \cdot e\{az \cos F\})] + Q(x)]); \quad (6b).$$

Here $U = r[F, x] \cdot a \cdot \cos F / \pi(a \cos F - qB)$; $B = e\{-h + S[n]a \cos F\}$. It is evident that ray's entry to objective coordinates depends on periods' number in trajectory. In every period ray performs straight

and reverse motions in axes X and Y , which amplitudes don't coincide with each other because of oscillations' frequency increasing in the following semicycle. That is the reason of coordinate's shift accumulating in periods.

Conclusions

1. Atmosphere in simulation model may be represented by optical system consisting of a number of 'plate-sphere' conjunctions.
2. Whole 'plate-sphere' conjunctions (its thickness equals to period) transform the going ray's parameters and remain entry angles' values.
3. Fractional 'plate-sphere' conjunctions transform parameters and going out rays.
4. The proposed model of the light propagation permits to realize atmospheric image's transformations in the level of physical laws and take into account variate RC disturbances and variations of medium's parameter's (the stratum's thickness, molecules' mass, temperature factor).

References

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