DETERMINATION OF THE ORIENTATION OF THE ACCRETION COLUMNS IN MAGNETIC CATACLYSMIC VARIABLES

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Abstract. Methods for the determination of the orientation of accretion columns in magnetic cataclysmic variables based on the observations of the circular and linear polarization, X-Ray flux and profiles of emission lines are discussed. The non-uniformity of the density distribution through cross-section of the accretion columns leads to significant changes in the resulting spectra in two polarizational modes as compared with the 'homogeneous plasma slab'model. Simplified expressions for the circular polarization provides an additional relation between the inclination i and the magnetic latitude β. Similar relations may be established from the X-Ray and spectral observations. However, by interchanging the values of i and β one will obtain the same phase relationship. The only direct method obtain the true values for i and β is based on polarization position angle changes. Real columns are non-stationary and not axisymmetric, which significantly affect observed characteristics. Observations of polars show long-term orientation changes of the accretion column with respect to the binary system. However, observations to date do not allow one to distinguish between the 'swinging' and 'idling' dipole models.

Key Words: Stars: Binaries: Magnetic: Accretion Columns
Stars: Individual: AM Her, EF Eri, MR Ser

INTRODUCTION. In 1976, S.Tapia (1977) discovered significant linear and circular polarization in the emission of AM Her. These polarization changes occur with the same ~3-hour period, as the photometric and spectral characteristics. Krzeminski and Serkowski (1977) found a similar phenomenon in AN UMa, thus arguing for the existence of a class of exotic objects, which they called 'polars'. According to the 'standard model' (Chanmugam and Wagner, 1977; Stockman, 1977 and references therein), these magnetic cataclysmic variables are binary systems consisting of a Roche lobe filling secondary and a magnetic white dwarf (WD) primary onto which accretion occurs via an accretion column (AC). The strong (20-70 MGs) magnetic field of the primary disrupts the formation of a disk. Reviews may be found in Kruszewski (1978), Chiappetti et al. (1980), Lamb (1985), Liebert and Stockman (1985), Voykhanskaya (1989), Aslanov et al.(1989) and Cropper (1991).

The strong magnetic field leads to a fast ($< 10^3$ yrs) synchronization of the rotation of the WD with the orbital motion (cf. Andronov,1987a). After such synchronization changes in the orientation of the magnetic axis of the WD with respect to the non-degenerate secondary may exibit a 'limit-cycle' behaviour as predicted by the 'Swinging Dipole' model (Andronov, 1987b). The changing orientation of the magnetic field in the vicinity of the inner Lagrangian point modulates the accretion rate (Andronov, 1984a) and thus the luminosity of the accretion column. Based on observations these changes were suspected by Bailey and Axon (1981), but the question concerning whether the WD is 'idling' or 'swinging' remains unsolved (cf. Andronov, 1992).

In this Paper we briefly discuss methods for the determination of the orientation, published earlier, and present the results of a simplified approach to the observations of the circular polarization and of the 'base' component of the emission lines.

INHOMOGENEOUS ACCRETION COLUMNS. We will use an AC which is tall and thin, despite the fact that models for some systems argue for a 'polar cap' rather than an AC. In previous papers (Andronov, 1990, 1992) we studied isothermal ACs with a non-uniform density distribution. To compute the fluxes in two independent polarizational modes we used the analytic approximation for the absorption coefficients derived by Pavlov et al.(1980). They suggested the homogeneous plasma slab, for which the intensity in the j^{th} polarizational mode is proportional to $r(\tau_j) = 1-exp(-\tau_j)$, where is the optical depth of the j^{th} mode. Thus, if for both polarizational modes $\tau_j >> 1$ the corresponding intensities are equal and no polarization may be observed. Similar results may be obtained for all models of the ACs with abrupt limits. Let the electron number density distribution be of the form

$$n(x,y,z)=n_0 f(x,y,z) \qquad (1)$$

where n_0 - is the characteristic density, and $f(x,y,z)$ is a dimensionless function of the coordinates. Here z-axis is directed along the AC's axis. The observer is located in the $y = 0$ plane.

Due to the finite mass of the AC, the integral

$$F(z)=\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y,z)\, dx\, dy \qquad (2)$$

must be finite. Other physical speculations about the structure of the stationary AC suggest that each curve of equal density

$$f(x,y,z) = const(z)$$

is a single closed line. For a symmetric AC this function may be rewritten in the form

$$f(x,y,z) = g(u,z), \quad (3)$$

where $u^2 = (x/a)^2 + (y/b)^2$, and a and b are the characteristic linear dimensions of the elliptic AC along the x- and y- axes at the height z.

If the function $g(u,z) > 0$ for all u, the inhomogeneity of the AC causes a significant dependence of the 'effective' AC's radius on the optical depth τ even if $\tau > 1$. The values

$$\tau = \chi_j n_0 f(x,y,z)\, dx, \quad (4)$$

where χ_j - are the absorption coefficient in the j^{th} polarizational mode (Pavlov et al., 1980). We studied the following two limiting cases:

$$g(u,z) = \exp(-u^2/2) \quad (5)$$

and

$$g(u,z) = 1/(1+u^2) \quad (6)$$

The first approximation corresponds to the electron number density which rapidly decreases with a distance from the AC's axis. The second one (cf. Wang and Frank, 1981; Stockman and Lubenow, 1987) makes the integral (2) infinite, thus such a model may be applied for the AC with the abrupt limit $u < u^0$. If $u > u^0$ then $g(u,z) = 0$. The corresponding expressions for the fluxes, as well as the computations of the polarization and spectra were published by Andronov (1990, 1992). In short, the 'effective radius' of the column $r(\tau)$ is proportional to τ for $\tau < 1$. For $\tau > 1$ the function $r(\tau)$ depends strongly on the shape of the function $n(x,y,z)$. For the homogeneous plasma slab, or homogeneous cylinder, with large $r(\tau) \approx 1$. For the 'normal distribution', $r(\tau) \approx (2 \log \tau)^{1/2}$. The largest slope corresponds to the 'Lorentz shape', see Eq.(6), where $t(\tau) \infty \tau$ while $\tau < u$. In the opposite case $\tau > u$ and the function $r(\tau) \approx u = const(\tau)$. The fluxes from real accretion columns are believed to depend on τ in an intermediate manner as compared with the limiting cases discussed above. Obviously, models may be significantly complicated since other effects may be taken into account. However, we did not compute the two-layer model like that proposed recently by Wu and Chanmugam (1988). The main polarizational effects are discussed in the monograph of Dolginov, Gnedin and Silantjev (1979).

MAGNETIC AXIS CROSSES THE CELESTIAL SPHERE. Despite the complicated dependence of the polarization and fluxes on the model parameters, some conclusions may be made from the qualitative analysis of the linear and circular polarization. The sign of the circular polarization depends on the angle between the line of sight and the magnetic field lines. Thus the linear polarization reaches its maximum simultaneously with the zero-crossing of the circular polarization, i.e. when the AC's axis lies orthogonal to the line of sight. Let α - is the angle between them, which may be obtained from the expression

$$\cos\alpha = \cos i \cos\delta + \sin i \sin\delta \cos(2\pi(t-t_0)/P) \quad (7)$$

where i is the inclination (i.e. the angle between the rotational axis and the direction to the observer) i, δ is the angle between the rotational axis and the AC's axis, t is the trial time, P_{orb} is the rotational period of the WD (coinciding with the orbital period), and t_o corresponds to the initial phase. Some authors prefer to use the 'latitudes' of the observer $(90^0 - i)$ and of the AC $(90^0 - \delta)$ (cf. Kruszewski, 1978).

One may see, that the Eq. (7) is symmetric with respect to the model parameters i and δ. Thus interchanging them will not change the dependence of $\alpha(t)$. Estimates of i and δ from each characteristic of the emission which depends only on α and not on the position angle does not allow one to obtain the true sequence of i, δ or δ, i in principle.

For $\alpha = 90^0$, $\cos\alpha = 0$, and

$$\varepsilon = \cot i \cot\delta = -cos(\pi\Delta\varphi), \quad (8)$$

where $\Delta\varphi$ is the phase difference between the zero-crossings of the circular polarization. Here there is an uncertainty in the sign of $\cos(\pi\Delta\varphi)$, because the values $\Delta\varphi$ and $1 - \Delta\varphi$ are both phase intervals in the same sense, but they correspond to the opposite magnetic poles of the WD. The sign depends on whether the active column and the observer are located on the same or opposite hemispheres of the WD. If they are on the same hemisphere, then the value on the right side of Eq.(8) must be positive.

From Eq.(8) one may not obtain both values i and δ, but may estimate the dependence of $\delta(i)$ or $i(\delta)$. Kruszewski (1978) obtained the value ε = -0.62 for AM Her, whereas Chanmugam and Wagner (1977) originally found ε = -0.68. Kruszewski (1978) also pointed out, that the values of ε are wavelength-dependent, thus arguing for possible differences in the effective orientation of the magnetic field in the region, effectively emitting at the corresponding wavelength.

POSITION ANGLE OF THE POLARIZATION: THE ONLY WAY TO DETERMINE THE INCLINATION? The position angle Θ of the linear polarization may be obtained from the expression

$$\cot\Theta = \cos i \cot\psi - \cot\delta \operatorname{cosec}\psi \sin i \quad (9)$$

where Θ is arbitrary set to zero, if $\psi = 0$. Generally, $\psi = 2\pi(t - t_\Theta)/P_{orb}$ (Chanmugam and Wagner, 1977; Stockman, 1977).

Efimov and Shakhovskoy (1981, 1982) studied phase changes of the linear polarization on the '$p_x - p_y$' diagram and estimated the orientation for the systems AN UMa and AM Her.

Meggit and Wickramasinghe (1982) pointed out, that

$$d\Theta/d\psi = \cos i \tag{10}$$

when the angle $\alpha = 90^0$ and the Eq.(8) is valid. They obtained $63^0 < i < 76^0$ for AM Her. According to Kruszewski (1978) i $= 65^o$ and $\delta = 143^o$ for AM Her ('far pole') or $\delta = 180^0 - 143^0 = 37^0$ ('closest pole'). Piirola et al.(1985) obtained from Eq.(8) the values $67^o > \delta > 53^o$ for V, and $61^o > \delta > 45^o$ for I, adopting $30^o < i < 46^o$. Barrett and Chanmugam (1984) and Brainerd and Lamb (1985) obtained the values $i = 46^0$ and $i = 35^0$ ($\delta = 58 \pm 5^0$) respectively. Piirola et al.(1985) fitted the position angle at all available phases by using Eq.(9).

Discrepances in the results may be partially explained by the insufficient accuracy of the position angle determination and its derivative. Another possibility is due to physical variability. However, in the latter case one has to suggest that the AC moves above the surface of the WD with a characteristic time-scale comparable with the orbital period. This is strange enouth because the rotation of the white dwarf is believed to be nearly synchronous (within one part in 10^{-4}) with the orbital motion.

DIFFERENT COLORS OF THE ACCRETION COLUMN: SIMPLIFIED CONSTRAINS FROM ECLIPSES. The polarizational phase curves are wavelength-dependent, as was originally found by Tapia (1977). This suggests that the cyclotron emission from the AC originates as a sequence of some first cyclotron harmonics (Gnedin and Sunyaev, 1973; Tapia, 1977). One may conclude, that each part of the column emits at different wavelengths, because the cyclotron frequency of the s^{th} harmonic is proportional to the magnetic field strength, i.e. $w \alpha s/r^3$ where r is the distance from the center of the dipole (Priedhorsky and Krzeminski, 1978). The part of the column at the distance r is eclipsed, if

$$\cos\alpha < - r_{WD}/r_i \tag{11}$$

where r_{WD} is the radius of the white dwarf. Combining (7) and (10), one may obtain the system of equations for each wavelength λ_i :

$$a + b\cos\pi\Delta_i = -(1 - (r_{WD}/r_i)^2)^{1/2} , \tag{12}$$

where Δ_i is the phase duration of the eclipse of the cyclotron emission at a given wavelength, $a = \cos i \cos\delta$, and $b = \sin i \sin\delta$. Here are three unknown parameters: a,b and r_1 (because for the dipole field approximation, the values

$$r_i/r_1 = (\lambda_i/\lambda_i)^{1/3} = (\omega_1/\omega_i)^{1/3} \tag{13}$$

are known). This system may be solved, if there are circular polarization curves obtained in at least three spectral bands.

Another possibility is to use an approximation for the circular polarization of the emission from the AC, the height of which is much more than the width (adopted from Gnedin and Pavlov (1975), cf. the expressions 5.111,5.112 of Dolginov et al. (1979))

$$q = 2(\omega_B/\omega)\cos\alpha \ |\sin\alpha|\gamma/(1 + |\sin\alpha|), \tag{14}$$

where ω_B is the cyclotron frequency , ω is the trial frequency, and γ is some coefficient. If one will use the trial frequency $\omega = s\,\omega_B$ for the s^{th} harmonic then, at the phase of the mid-eclipse,

$$sq_{0i} = \frac{2(a+b)\sqrt{1-(a+b)^2}\,\gamma_i}{1+\gamma_i\sqrt{1-(a+b)^2}} \tag{15}$$

whereas at the phase 0.25 earlier or later (practically one has to use the mean value),

$$sq_{0.25i} = \frac{2a\sqrt{1-(a+b)^2}\,\gamma_i}{1+\gamma_i\sqrt{1-a^2}}. \tag{16}$$

The inverse equations are

$$\gamma_i = \frac{sq_{0i}}{(2(a+b) - sq_{0i})\sqrt{1-(a+b)^2}}$$

$$\gamma_i = \frac{sq_{0.25i}}{(2a - sq_{0.25i})\sqrt{1-a^2}} \tag{17}$$

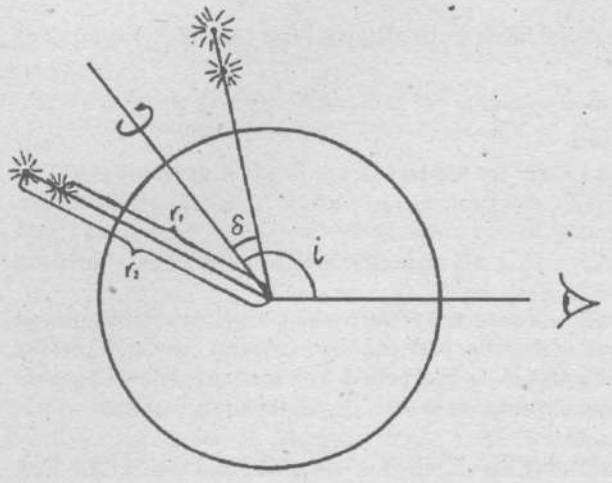

Fig. 1 The schematic model of the white dwarf and the accretion column in MR Ser. Upon suggestion, each part of the accretion column emits radiation at the cyclotron frequency $s \cdot \omega_B$, corresponding to the magnetic field strength B at the distance r.

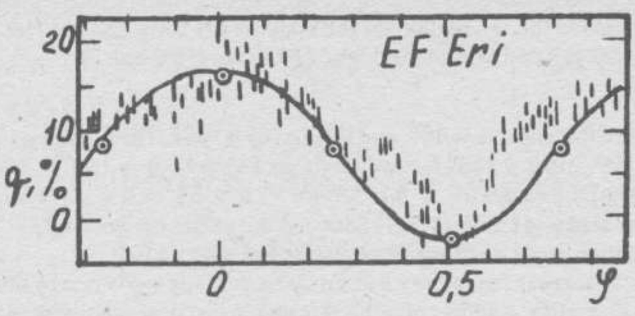

Fig. 2 The circular polarization changes of MR Ser: crosses are original observations of Liebert et al.(1982); filled circles are the adopted points corresponding to the values of q_0 and $q_{0.25}$; the solid line is the calculated (Eq.(14)) changes with eclipse by the white dwarf; the broken line - the calculated changes without eclipse.

If we have observations in two spectral bands, than, varying r_1, one may consecutively obtain r_2 (Eq.13), a, b (Eq.12). The value r_1 must satisfy two eq.(17) for the same values of $\gamma_1, \gamma_2...$, (they may be slightly different for the different spectral bands).

Using this algorithm for the observations of MR Ser=PG1550+191 (Liebert et al., 1982), we obtained for s=1 the values $i = 56^\circ \pm 6^0$ ($124^\circ \pm 6^0$ for the 'far pole' model), $\delta = 23^0 \pm 6^0$, $\gamma = 0.12 \pm 0.01$, $r_1 = (1.24 \pm 0.08)\ r_{WD}\,(0.407\mu m)$, $r_2 = (1.50 \pm 0.10)\ r_{WD}\,(0.727\,\mu m)$.

The eclipse durations $\Delta_1 = 0.39 \pm 0.02$, $\Delta_2 = 0.29 \pm 0.02$. The deviation of our values of i and δ from that ($i = 45^0 \pm 5^0$, $\delta = 40^0 \pm 5^0$) of Brainerd and Lamb (1985) do not exceed 3σ. The scheme of the system is shown in Fig.1. The corresponding curves are shown in Fig.2, as well as the positions of the eclipses. Because the region of the emission in the given spectral band is an extended one, the observed eclipses are not abrupt. It may be noted that we used only four points from the two curves (marked by pluses), but the curve fitting is satisfactory despite so many assumptions.

For AM Her we used the observations of Bailey and Axon (1982) with the spectral bands centered at 0.55, 0.68 and 0.83μm, and the 'far pole' model of Kruszewski (1978). The eclipse durations are Δ_i = 0.40, 0.37 and 0.34 (± 0.01), respectively. The results are the following: $\gamma = 0.110 \pm 0.015$, $i = 67^0 \pm 6^0$, $\delta = 119^0 \pm 6^0$, $r_1 = (1.21 \pm 0.07)\,r_{WD}$, $r_2 = (1.30 \pm 0.12)\,r_{WD}$, and $r_3 = (1.39 \pm 0.09)\,r_{WD}$.

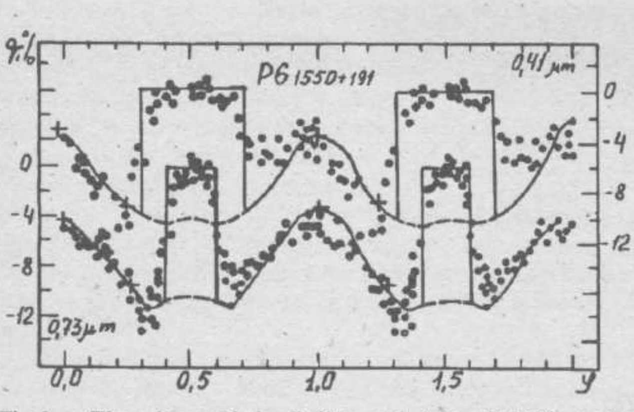

Fig. 3 The observed, by Bailey and Axon (1981), and calculated changes of the circular polarization of AM Her. The legends are as in Fig.2.

The corresponding curves are shown in Fig.3. The values of i and δ are similar to that ($i = 65^\circ$, $\delta = 143^\circ$) of Kruszewski (1978). This 'far-pole' model may not contradict the 'nearest pole' model of Brainerd and Lamb (1985), Piirola et al. (1985) and Andronov (1986b), if there are two simultaneously accreting poles which are emitting at different spectral bands (Kruszewski, 1978). Both poles outside the eclipses will give the same phase dependence of $\cos\alpha$ and Θ, the inclination of the column(s) with respect to the white dwarf may lead to the discrepancy in δ.

For the one-band observations of the systems without eclipse of the accretion column, the orientation may be determined by using the values $q_0, q_{0.25}$ and $q_{0.5}$ and substituting them into Eq.(14). For observations of EF Eri (Bailey et al., 1980), we obtained $i = 75^0$, $\delta = 18^0$ (or opposite sequence) and $\gamma = 0.218$ suggesting s = 1. Polarization curve is shown in Fig. 4.

These numerical solutions show, that the parameter γ is constant within observational errors for the fixed star, but different for different spectral bands. With changing s, the parameters i and δ are changing not significantly, contrary to γ. From theoretical considerations, the values of γ must lie in the interval from 0.30 to 1.57 (Gnedin and Pavlov, 1975). For AM Her, this corresponds to s=3,4,5 or 6. The corresponding magnetic field strength at the pole of the white dwarf is B=50 to 100 MGs, intermediate between the values 260 MGs (Voykhanskaya

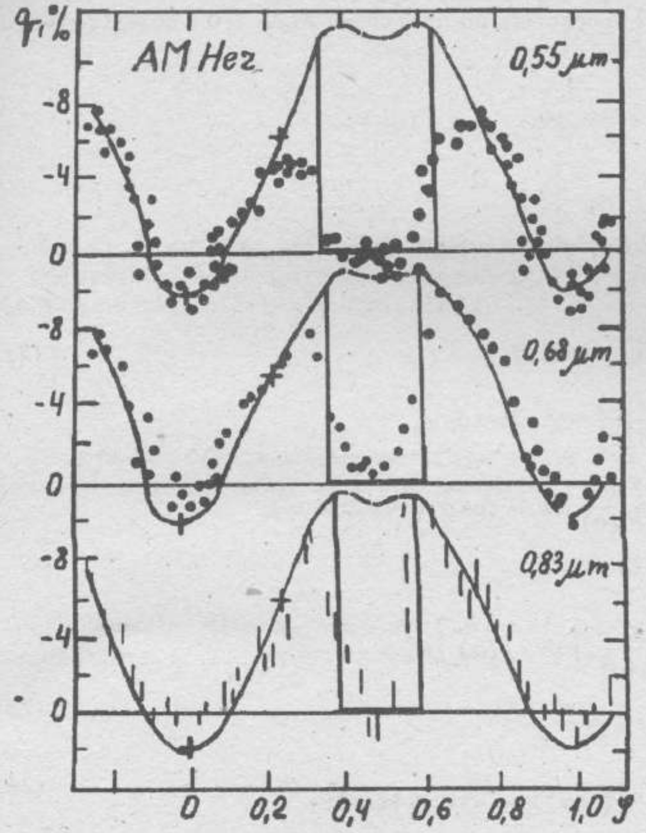

Fig. 4 The observed, by Bailey et al. (1980), and calculated changes of the circular polarization of EF Eri. The legends are as in Fig.2, with an additional point corresponding to $q_{0.25i}$.

and Mitrofanov, 1980) and 20 MGs (Schmidt et al., 1981). The dilution of the cyclotron emission by the contribution of the stellar components leads to a decrease of the value of q, thus to a decrease in the 'observed' value of γ and an increase in s.

This model is very crude. In principle, one must take into account the true three-dimensional functions of the distribution of the electron number density, velocity and the magnetic field. However, estimates of i and δ from the eclipse width may give the additional information for model constrains.

X-RAY CURVES: THE 'POLAR CAP' MODEL. Besides the polarimetric observations, the orientation may be estimated from the soft X-ray curves. Andronov (1986b) computes a set of models for the beamshape of soft X-ray emission produced in an axially symmetric 'polar cap'. No 'optically thin' model gives a satisfactory approximation to the observations of AM Her (Hearn and Richardson, 1977). Therefore the extinction corresponding to the density distribution (5) is taken into account. For some values of the model parameters, the following ranges are obtained : $51^0 < i < 64^0$, $30^0 < \delta < 34^0$ (or opposite), in exellent agreement with the mentioned above 'near pole' model.

The soft X-Ray flux will be re-emitted in the bottom parts of the AC, thus the region of its origin will not be two-dimensional, and will extend to some height above the surface of the white dwarf. This region will be 'boiling', resulting in rapid variability in a time-scale of seconds (Andronov, 1987c).

EMISSION LINE PROFILES: THE WINGS ORIGINATE CLOSE TO AC ? The optical spectra of polars are characterized by the presence of the strong emission lines of hydrogen and helium, the shape of which is very complex and variable (cf. Cowley and Crampton, 1977). According to Burenkov and Voykhanskaya (1980) during the low luminosity state the lines have many peaks, which argues for the motion of separate plasma blobs rather than a homogeneous accretion flow. With increasing luminosity the lines become smoother, indicating the formation of an accretion column (AC).

Following Andronov (1984b), let us consider the profile of the emission line arising at the AC near the magnetic pole of the white dwarf. Let $f(v/v_0, \alpha, r)\,dv\,dr$ be a flux at the wavelength corresponding to the velocity v in the co-moving reference frame which appears from the element of AC at the distance r from the center of the white dwarf. Here $v_0(r)$ - is the characteristic velocity, which affects the line width, and α - is the angle between the line of sight and the AC's axis (directed to the center of the white dwarf). In the reference frame related to the white dwarf, one may write the general expression

$$F(v,\alpha) = \int_r dr \cdot f((v - v_r \cos\alpha)/v_0, \alpha, r). \tag{18}$$

The luminosity of the AC's element is

$$L(\alpha,r) = \int_{-\infty}^{\infty} f(v/v_0, \alpha, r)\,dv. \tag{19}$$

The total luminosity is equal to

$$\int_r L(\alpha,r) = \int_{-\infty}^{\infty} F(v,\alpha)\,dv. \tag{20}$$

Let us introduce the function

$$F_n(\alpha) = \int_r dr \int_{-\infty}^{\infty} v^n f(v/v_0, \alpha, r)\,dv. \tag{21}$$

The function $f(v/v_0, \alpha, r)$ is symmetric with respect to the argument v. This is valid while suggesting the Doppler effect, Zeeman effect, and the collisional broadening. Thus, for integer values of m, one may obtain $F_{2m+1} = 0$. The mean values of are the following:

$$<v^n> = \frac{1}{F_0(\alpha)} \int_{-\infty}^{\infty} v^n (f\,(v - v_r \cos\alpha)/\,v, \alpha, r)\, dv =$$

$$= \frac{1}{F_0(\alpha)} \int_r dr \int_{-\infty}^{\infty} (v_1 + v_r \cos\alpha)^n f(v_1/\,v_0, \alpha, r)\, dv_1\,,$$

$$<v> \; = \; <v_r> \cos\alpha \tag{22}$$

$$<v^2> = \frac{F_2(\alpha)}{F_0(\alpha)} + <v_r^2> \cos^2\alpha,$$

$$<v^3> = 3\,\frac{F_2(\alpha)}{F_0(\alpha)} <v_r> \cos\alpha + <v_r^3> \cos^3\alpha$$

Here

$$<v_r^n> = \frac{1}{F_0(\alpha)} \int_r dr \cdot L(\alpha, r)\, v_r^n. \tag{23}$$

The mean-squared thickness of the line is

$$M_2 = <v^2> - <v>^2 = \frac{F_2(\alpha)}{F_0(\alpha)} + (\,<v_r^2> - <v_r>^2\,) \cos^2\alpha\,. \tag{24}$$

The asymmetry is proportional to the quantity

$$M_3 = <v^3> - <v>^3 - 3\,M_2 <v> =$$

$$= (<v_r^3> - 3 <v_r^2> <v_r> + 2 <v_r>^3\,) \cos^3\alpha\,. \tag{25}$$

Thus, the asymmetry of the lines appears to be due to the velocity gradient of the plasma motion. If the magnetic axis crosses the celestial sphere then the asymmetry changes its sign. Simultaneously the mean-squared thickness reaches its minimum. Otherwise these characteristics are changing in phase (or in the opposite phases).

The orbital changes of M_2 and M_3 are determined by the orientation of the AC in the rotating frame. Taking into account the expression (7), one may rewrite (24) and (25) in the form:

$$M_2 = A^2 + \widetilde{B}^2 \cos^2\alpha \tag{26a}$$

$$M_3 = C\,(D + E\cos\Psi)^3 \tag{26b}$$

where

$$C = (<v_r^3> - 3 <v_r^2> <v_r> + 2 <v_r>)\, B^{-3}.$$

If the function $L(\alpha, r)$ may be expressed as the product $L_1(\alpha) \cdot L_2(r)$, the parameters A, $\widetilde{B}$, and C do not depend on α. By using the method of the Least squares, and solving the system of the equations

$$a_0 + a_1 \cos\Psi_k + a_2 \sin\Psi_k + a_3 \cos 2\,\Psi_k + a_4 \sin 2\,\Psi_k = M_{2k},$$

$$k = 1 \ldots N \tag{27}$$

(where N - is the number of the observations), one may obtain the coefficients $a_0, \ldots, a_4$. They are related to the model parameters :

$$A^2 + F^2 + \frac{1}{2} G^2 = a_0,$$

$$2FG\cos\Psi_0 = a_1,$$
$$2FG\sin\Psi_0 = a_2,$$
$$\frac{G^2}{2}\cos 2\Psi_0 = a_3,$$
$$\frac{G^2}{2}\sin 2\Psi_o = a_4. \tag{28}$$

Here $F = \bar{B}\cos i\cos\delta$ and $G = \bar{B}\sin i\sin\delta$.

The system of 5 equations with 4 unknowns (A, F, G, Ψ_0) may be reduced to 2 equations with 3 parameters$(\bar{B}, i, \delta)$ by obtaining the values of F and G from (28), i.e.,

$$G = \sqrt{2\sqrt{a_3^2 + a_4^2}} \quad ,$$
$$F = \pm\sqrt{a_1^2 + a_2^2}\ /2G. \tag{29}$$

The sign of F is to be choosen so that Ψ_0 will be close to the phase of the minimum radial velocity. Thus the spectral observations alone may allow one to write the relation

$$\cot i\ \cot\delta = F/G \tag{30}$$

Substituting the values of F and G into (26.b), one may estimate the value of C . The parameter C allows one to estimate the dependence of $L(\alpha, r)$ on r. For $L(\alpha, r)\,\alpha\, r^{-1/2}$, and $v_r\ \alpha\, r^{-1/2}$ (free fall approximation), one may obtain

$$\frac{<v_r>}{\bar{B}} = 2\sqrt{\beta(\beta-1)}\ ,$$
$$C = \frac{\beta-1}{\beta+\frac{1}{2}} - \frac{3(\beta-1)^2}{\beta(\beta-\frac{1}{2})} + \frac{2(\beta-1)^3}{(\beta-\frac{1}{2})^3}\ ,$$
$$\bar{B} = \frac{v_0}{2\beta-1}\sqrt{\frac{\beta-1}{\beta}}\ . \tag{31}$$

Here β is the model parameter, which may be calculated from (31) by using the value of C . The next step is to obtain $<v_r>/\ \bar{B}$. The function $C(\beta)$ is monotonically decreasing with increasing β , crossing zero at $\beta = 1.5$.

The distance is related to the velocity, thus the dummy variable r may be substituted by v_r. If v_r is proportional to the free-fall velocity,

$$L(\alpha, r)\,dr\ \propto\ L(\alpha, r_o v_0^2/v_r^2)\frac{dv_r}{v_r^3}. \tag{32}$$

Here v_0 - is the maximum radial velocity corresponding to the distance r_0 . Thus if the function $L(\alpha, r)$ decreases faster than $r^{-1.5}$, the major contribution would correspond to the high-velocity regions (otherwise to the low-velocity regions). The broadening will smooth the line, but qualitatively will not affect it.

The observed radial velocity is

$$v_{obs} = \gamma + v_{orb}\sin i\ \sin(\Psi-\Psi_1) + \frac{<v_r>}{\bar{B}}(D + E\ \cos(\Psi-\Psi_0))\,. \tag{33}$$

Here γ corresponds to the center of mass, $v_{orb}\sin i$ - is the projection of the velocity of the white dwarf onto line of sight. From the sinusoidal fit to v_{obs} , one may extract the contribution of the orbital motion of the white dwarf. The angle between the projection of the AC's axis onto the orbital plane and the line of centers is $\Delta\Psi = \Psi_1 - \Psi_0$.

Thus the mean-squared width of the line is at minimum if the angle α is close to 90, or is at maximum (the hottest regions are eclipsed by the white dwarf). This prediction agrees qualitatively with the observations of AN UMA (Schneider and Young, 1980), because the X-Ray minimum coincides in phase with the minimum of the relative flux and the full width at half-maximum and the maximum of the circular polarization. This may be interpreted as the eclipse of the emission in X-Rays and in spectral lines (the minimum of the continuum light appears slightly earlier).

However, for the quantitative interpretation, a detailed study of the phase dependencies of the observed characteristics of the emission lines are needed. The comparison of the observations with the model computations may allow one to detect other possible sources of the emission lines (e.g. the accretion flow and the heated part of the atmosphere of the secondary). If the lines have their origin near the accretion column, the derived expressions may allow to determine its orientation.

ROTATIONAL EVOLUTION OF THE WHITE DWARF: 'SWINGING' OR 'IDLING' ? The orientation of the accretion columns with respect to the white dwarf underwent changes, probably of cyclic character, which one may see from the photometric and polarimetric observations of AM Her, QQ Vul and apparently some other polars (Andronov, 1987a). For AM Her, the values of δ vary from 28^0 to 41^0 ($i=64^0$), or from 58^0 to 69^0 ($i=35^0$), i.e. with the total amplitude $11-13^0$ (Shakhovskoy et al., 1992; Andronov, 1992). The variations of the 'longitude' Ψ_p in AM Her has $\approx 28^0$ which is twice as large (Andronov, 1987a and references therein). However, the number of the available times of sign reversal of circular polarization is not sufficient to obtain the $\Psi_p(t)$ and $\delta(t)$ curves, and to choosen between the model of the 'swinging' (Andronov, 1987a) or 'idling' (Campbell, 1983) dipole. Thus regular polarizational observations of polars are needed.

The problem of the rotational evolution of the white dwarf is closely linked with the problem of the long-term brightness variations of polars. Since the time of discovery of 'high' and 'low' states by Hudec and Meinunger (1976) the following models were proposed:

a) changes of the irradiation in the vicinitiy of the inner Lagrangian point by the emission from the compact primary and subsequent changes of the accretion rate (Basko and Sunyaev, 1973; King and Lasota, 1984);

b) changes of the accretion rate due to the 'Magnetic Valve' mechanism (Andronov, 1984a), i.e. the modulation by the changing orientation of the magnetic field in the vicinity of the inner Lagrangian point;

c) changes of the accretion rate due to the solar-type activity of the secondaries (Bianchini, 1990), for which the changes are often abrupt ('switchings'), rather than quasi-sinusoidal (Andronov and Shakun, 1990); possible minor changes of the distance between the stars due to the presence of the planet-like third body (Andronov, 1992);

d) 'switchings' of the accretion from one pole to another during 'swingings' or 'idlings'. If the column is located at the 'far side' of the white dwarf, an 'inactive state' may be observed even if the accretion rate is not reduced.

THE BOTTOM PART OF THE ICEBERG: THE COLUMNS ARE INCLINED, ELLIPTIC, THICK, BOMBARDED, BOILING, DOUBLE ... The above discussed approximations correspond to the case of axially symmetric columns. However, in real objects, ACs may significantly deviate from the symmetric shapes and thus disturb the phase curves of polarization and fluxes. For the stationary AC, one has to take into account the following primary effects: a) the inclination of the column; b) its ellipse-like shape; c) deviation from the self-similarity.

The inclination is important for the bottom parts of the column, which may be eclipsed at certain phases. The effect reduces with increasing height in the AC (Andronov, 1986a). The ellipticity of the cross-sections of the AC may lead to an additional distortion of the phase curves if the AC is optically thick for cyclotron emission (Andronov, 1992). The third effect must be computed numerically from the self-consistent equations of the column's structure and emission.

In the above mentioned models, the height of AC was assumed to be much larger than its width. This is possibly the case for the 'true' polars (AM Her type), whereas for the intermediate polars (DQ Her type), these dimensions are more similar (cf. Chanmugam and Frank, 1987; Canalle and Opher, 1991).

The problem becomes much more compicated if accretion occurs onto both poles and there are two columns with independent parameters. Such models for AM Her itself were discussed by Kruszewski (1978), Piirola et al.(1985) and Wickramasinghe et al. (1991). If the dipole is shifted from the center of the WD then the magnetic field may be of different strength and direction in two columns. Such an 'offset dipole model' may lead to the revolutionary changes in the 'standard' model of polars, where the magnetic field is believed to be strong enough to determine the plasma motion from the vicinity of the inner Lagrangian point. If two columns are existing simultaneously, without 'switchings' then the radius of the magnetosphere must be much smaller as previously suggested. In this case, the angular momentum transfer must be redetermined, because the synchronization of the rotation of the white dwarf with the orbital motion seems to be very strange.

The stationary models are only the intermediate approximations. The accretion stream is not homogeneous, the separate 'blobs' become long and thin while moving to the white dwarf, thus the AC is 'bombarded' by 'spaghetti' (Panek, 1980) and may not be stationary. Even for homogeneous accretion flow, the AC's structure may undergo drastic changes on a time scale of seconds (Langer et al., 1982). Such ultrarapid QPOs may arise from single 'spaghetties' and last for some dozens of seconds. Such possible exotic types of instability, as the 'boiling', 'tornado' or 'beacon' columns, may be excited (Andronov, 1987c).

Despite the corresponding fast variations are usually smoothed during the observations, the polarization and spectra computed for the 'stationary' and 'non-stationary' AC may differ significantly. Thus 3-D time-dependent models need to be computed, despite the fact that the number of parameters which need to be determined is quite large.

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